Socio-Economic Planning Sciences 40 (2006) 212–248
SOCIO-ECONOMIC PLANNING SCIENCES
www.elsevier.com/locate/seps

A multiobjective optimization approach to smart growth in land development

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Available online 4 May 2005

Abstract

In this paper we apply a multiobjective optimization model of Smart Growth to land development. The term Smart Growth is meant to describe development strategies—that do not promote urban sprawl. However, the term is somewhat open to interpretation. The multiobjective aspects arise when considering the conflicting interests of the various stakeholders involved in land development decisions: the government planner, the environmentalist, the conservationist, and the land developer. We present a formulation—employing linear and convex quadratic objective functions subject to polyhedral and binary constraints for the stakeholders. The resulting optimization problems are convex, quadratic mixed integer programs that are NP-complete. We report numerical results with this model for Montgomery County, Maryland, and present them using a geographic information system (GIS).

Keywords: Multiple objective programming; Integer programming; Quadratic programming; Smart Growth; Large Scale Optimization; Geographic Information System; Land use planning

1. Introduction

Currently, in land development, there is a move towards intelligent stewardship of natural resources to avoid urban sprawl. Such development schemes are often called Smart Growth.

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0038-0121/$ - see front matter
doi:10.1016/j.seps.2005.02.001

However, this term can be a bit nebulous since what constitutes Smart Growth for one stakeholder (or stakeholder group) may not necessarily be intelligent management of resources for another stakeholder. One needs to consider land development with all stakeholders' interests taken into account.

Smart Growth may be thought of as the present day incarnation of what was referred to as-growth management in the 1970s and 1980s [1]. Often, Smart Growth is defined in terms of what it seeks to accomplish. For instance, the Urban Land Institute defines this term as "Smart growth … enhances the economy, protects the environment and preserves or improves a community's way of life." [2]. While this definition makes Smart Growth appear desirable, it does not shed sufficient light on how such growth can be measured. More quantifiable definitions are offered by Downs [3]; these definitions involve the conservation of open space, mixed use development convenient to pedestrians and transit access, re-development of aging urban or inner suburban areas, and the use of boundaries to limit new growth. In the spirit of Downs' approach towards quantifiable measures, we define Smart Growth as a development plan leading to the Pareto optimal value of precisely defined measures identified by stakeholders.

Governments have relied on a host of tools to foster socially beneficial land development along the lines of Smart Growth, if not explicitly stated. Such tools include: public ownership, regulatory or educational approaches, or methods that contain financial incentives/disincentives [4,5]. Areas of application include: public ownership of parks, greenways, development impact fees, brownfields redevelopment, minimum density zoning, annexation, and purchase or transfer of development rights [4,6–8]. Unfortunately, these approaches do not account for the goal of each stakeholder with explicit objective functions. Furthermore, they fail to arrive at a compromise solution that embodies the zero sum nature of the process, a common case given their often diametrically opposing views (e.g., develop vs. conserve the land).

To model explicit tradeoffs between stakeholders in this zero-sum setting, as presented in [9], we consider four main classes of stakeholders: The government planner, The environmentalist, The conservationist, and The land developer. The resulting mathematical formulation is a multiobjective optimization problem, which is restricted by general constraints such as land growth rates and zoning, whose objectives correspond to each of the stakeholders' interests. Together with the work in [9], this multiobjective and novel approach applied to Smart Growth allows regional planners and other interested parties to balance the tradeoffs between the competing stakeholders.

Unlike the case of single objective optimization in which the total system cost or other system attribute is optimized, a different notion of Pareto optimality, is needed. A Pareto optimal solution to a multiobjective optimization problem means that an improvement in one of the objectives must come at the expense of at least one of the other objectives [10–12]. In the current Smart Growth setting, a Pareto optimal point corresponds to a particular development plan for the land parcels under consideration.

Over the years, models have been developed to study issues in land development, urban growth, or more recently Smart Growth. This is a complex area to analyze, involving many disciplines. Several approaches have been applied with varying degrees of detail. Thus, the choice for the Smart Growth modeler is to select just the right amount of detail to capture the inherent tradeoffs between the different stakeholders and between the system constraints. Too much detail, while providing a more realistic model, may make the model computationally intractable, and could potentially create more narrowly defined, and therefore less useful, results. Too stylized a model, on the other hand, while more generally applicable and more computationally tractable, might not provide results that capture the tradeoffs between stakeholders. We believe our approach, while retaining some crucial details, is a compromise between these two extremes. In particular, enough detail is used to capture these tradeoffs, yet the model is computationally tractable.

Other non-optimization methods in land use planning do not always model the behavior of the stakeholders to measure explicit tradeoffs between them. Other detailed modeling approaches in land use planning include: statistical forecasting of land use based on historical trends [13,14], Monte Carlo simulation using population growth and other factors to gauge quality of life [15], cellular automata where simple rules are applied between neighboring cells to simulate spatial development of a geographic area [10], and geographic information systems (GIS) and remote sensing to view urban change from a visual perspective [16]. While these approaches can be detailed and informative, any tradeoffs are not normatively based. That is, these approaches do not directly optimize development from each stakeholders' perspective and then arrive at a Pareto optimal compromise in the interests of all parties involved, as is done with our approach.

Other authors have also considered general land development problems from the multiobjective optimization perspective. These works differ in the specific problem formulations being studied, as well as in the solution methodologies employed. Formulations involving integer restrictions and other nonconvexities have often used heuristic methods because of the computational complexities involved. In these cases, enumeration of the entire Pareto optimal set, while possibly desirable, is computationally challenging. In the current work, for each set of weights used to find a Pareto optimal point, we solve a quadratic, mixed integer program. For the Montgomery county region, this perspective resulted in about 3500 variables (mostly binary) and over 23,000 constraints. Finding a subset of Pareto optimal points rather than the entire set of solutions illustrates the significant tradeoffs between the various stakeholders, which involve conservation of the environment, protection against urban sprawl, and economic benefits. In what follows we briefly review some selected multiobjective optimization works related to land development.

Bammi and Bammi's [17,18] early papers in multiobjective land development presented a multiobjective optimization model for land use planning in DuPage County, Illinois. They used a weighted-objectives approach that considered adjacent land uses, travel time, tax costs, negative environmental impacts, and costs borne by the community. Using a linear programming model for each of 147 planning regions, they computed acreage totals by land use type, which were then allocated by planners on a parcel-by-parcel basis. Later, Wright et al. [19] considered a multiobjective integer programming model for land acquisition. These integer restrictions greatly complicated the solution methodology and the authors developed a specialized approach given the possibility of gap points [20], i.e., solutions, which could be missed when using a weighting method. Their model considered three objectives: area of a cell, acquisition cost, and compactness of the developed cells. The largest problem they considered involved 30 cells which had 146 binary variables and 69 constraints. At that time this problem was limited by general-purpose multiobjective integer programming algorithms. Benabdallah and Wright [21] later extended various parts of this work. Next, Gilbert et al. [22] developed a four-objective optimization model that also contained integer restrictions on the variables. Their objectives included: the acquisition and development cost, the amenity and detractor distances, and the shape objective. Due to their formulation's computational complexity, they developed an interactive, partial enumeration scheme. This method was applied to solve land development plans for Norris, Tennessee, represented by 900 cells of approximately 2.5 acres each. More recently, the book edited by Beinat and Nijkamp [23] described a good collection of multiobjective land use papers with GIS components. Lastly, the recent work by Moglen et al. [9] considered a multiobjective integer programming problem using 810 parcels in Montgomery County, Maryland. The positions of four stakeholder classes—environmentalist, conservationist, government planner, and land developer—were considered in combination with certain global constraints, such as growth rates by each of five land use zones.

The current work extends [9] in several important ways and provides more of a mathematical perspective. First, using a different database of 913 undeveloped and 4837 developed parcels for Montgomery County, Maryland, the current work includes specific integer constraints to classify unassigned parcels into one of the five zones: residential low density, residential medium density, residential high density, commercial, or industrial. The previous work used a heuristic to assign unclassified parcels—so-called Rural Density Transfer—to one of these zones prior to running the optimization. All parcels 500 m from main roads were assigned an industrial land use consistent with land use elsewhere in the study region. Remaining patches of rural density transfer zoned areas were assigned one of the other land use categories in an ad hoc manner; thus, re-assigned parcels took on the same zoning category as nearby parcels already zoned in that category.

In contrast, the current work, where the model chooses which zone is appropriate for each unassigned parcel, is more efficient from the land use perspective but does represent a computational challenge. For example, for each of these 512 unassigned parcels, five additional binary variables (one for each of the zones) need to be included. Second, the current work, unlike [9], also includes a set of constraints to ensure that these unassigned parcels are only selected when necessary, with the preference given to parcels already classified into one of the five zones. Third, the current work, also unlike [9], considers the compactness of the developed area as an objective for the government planner. All else being equal, a more compact area is better from the perspective of the government planner since it means that less infrastructure (e.g., roads, water distribution network) is needed. Other works [19,22] have considered compactness in a variety of ways that we extend in the current work. The current approach minimizes the diagonal of the outer rectangle of the developed parcels and results in a convex quadratic objective function. The resulting optimizations from the weighting method [24] are generally instances of large-scale, quadratic mixed integer programs (QMIPs). The class of QMIPs is NP-complete [25]. However, the relaxed version of these QMIPS are simply convex, quadratic programs with linear constraints and thus represent a reasonable computational burden given the state of the art in optimization solvers. Our novel approach creates a reasonable balance between representing compactness and computational considerations.

The rest of this paper is organized as follows: Section 2 presents the Smart Growth model; Section 3 provides several theoretical results for the multiobjective formulation; Section 4 describes selected numerical results for Montgomery Country, Maryland; and Section 5 summarizes our findings.

2. The Smart Growth multiobjective optimization problem

To fairly represent the land development process, we model the objectives of four main stakeholder groups with competing objectives: government planners, environmentalists, conservationists, and land developers. At present, competing stakeholder objectives are not considered in most Smart Growth designs. Instead, a range of best management practices might be used. Examples include: incorporating porous pavement, rain gardens, or grassed swales [26] in an effort to minimize the impact of development. Rigorous comparisons of multiple alternative development patterns are generally not considered either. In fact, Smart Growth may be made more complicated by the advocates of this strategy. Balancing the interests of the diverse stakeholders from a multiobjective optimization perspective involves some sort of compromise strategy that can be analyzed over many different time periods. The current work considers a snapshot of the tradeoffs between these stakeholders.

2.1. An overview of the multiobjective optimization model

Multiobjective optimization problems can be stated as

$$\begin{array}{ll} \min & \{f_1(x), \ldots, f_k(x)\} \\ \text{s.t.} & x \in F, \end{array} \tag{1}$$

where $f_1, \ldots, f_k$ are given (real-valued) objective functions defined on some feasible region $F \subseteq R^n$. When the preferences of the ultimate decision-maker are not stated, generating methods are used to obtain Pareto optimal points to (1). Two common techniques employed to obtain elements of this Pareto optimal set are: the weighting method and the constraint method [24]. With the former approach, positive weights are applied to each objective and the sum of these weighted objectives is then minimized subject to the original feasible region F. A solution of this weighted subproblem is a Pareto optimal point of the original multiobjective optimization [11, Theorem 3.1.2]. Pareto optimal points can be generated by appropriate choice of these weights,. However, no preferences are imputed with this method. That is, the weights do not correspond to how much each objective is valued; instead, the weights are a mechanism to obtain Pareto solutions. The weighted subproblem is given as

$$\begin{array}{ll} \min & \sum_{i=1}^{k} w_i f_i(x) \\ \text{s.t.} & x \in F, \end{array} \tag{2}$$

where $w_i > 0,\ i = 1, \ldots, k$.

In contrast, the constraint method arbitrarily chooses one of the objectives to optimize, creates additional constraints that bound the values of the other objectives, and solves the resulting single objective problem. At optimality, if these additional constraints are binding, then the solution corresponds to a Pareto optimal point [24]. Iterative procedures for either of these two approaches can be used to generate a reasonable approximation to the Pareto optimal set, which can then be analyzed by the decision-maker. Certain duality gap points can exist with the weighting method if the Pareto optimal set is not convex. As a result, certain Pareto solutions cannot be obtained by solving an appropriate weighting problem. These solutions, however, can be found by the constraint method, but sometimes with a large computational burden [20]. In spite of duality gaps, the weighting method is a simple approach to generate different Pareto optimal solutions [11,27] (i.e., a grid of weights). Moreover, if enumerating only a representative subset of the entire Pareto optimal set is the goal, as in this study, the weighting method is reasonable.

For the multiobjective optimization problem concerning Smart Growth, we first designate S as the set of indices for each of the parcels of land that might be developed. For a typical parcel $i \in S$

$$d_i = \begin{cases} 1, & \text{if parcel } i \text{ is developed,} \\ 0, & \text{otherwise.} \end{cases}$$

Thus, fractional development of a parcel is not allowed. Since there are four stakeholder groups considered, the number of objectives is $k = 4$. Additionally, for computational reasons, we consider only linear constraints with some binary variables (e.g., d_i). Therefore, the form of the multiobjective optimization is

$$\begin{aligned} \min \quad & \{f_{\mathrm{GP}}(x), f_{\mathrm{E}}(x), f_{\mathrm{C}}(x), f_{\mathrm{LD}}(x)\} \\ \text{s.t.} \quad & Ax \leqslant b, \end{aligned} \tag{3}$$

$$\begin{aligned} & x_i^1 \in \{0, 1\}, \quad i = 1, \dots, n_1, \\ & x_j^2 \in [l_j, u_j], \quad j = 1, \dots, n_2, \end{aligned}$$

where $f_{\mathrm{GP}}(x), f_{\mathrm{E}}(x), f_{\mathrm{C}}(x), f_{\mathrm{LD}}(x)$ are, respectively, the objective functions for the government planner, the environmentalist, the conservationist, and the land developer; $x = \binom{x^1}{x^2}$ is the set of n decision variables broken out into n_1 that are binary (x^1) and n_2 that are bounded above and below by the vector of lower bounds l and upper bounds u and specified as (x^2). Lastly, the linear inequalities $Ax \leqslant b$ signify important system constraints that must be met (e.g., zoning growth patterns) as part of this multiobjective optimization.

There are many reasonable choices for the objectives $f_{\mathrm{GP}}(x), f_{\mathrm{E}}(x), f_{\mathrm{C}}(x), f_{\mathrm{LD}}(x)$. For example, the government planner could seek to confine development in prescribed growth areas, minimizing the extent to which new infrastructure can be built. A similar statement applies to the other stakeholders. Also, there is some degree of flexibility in the constraints in (3).

Independent of the level of detail, the government planner is trying to maximize the benefit to society at large by selecting high value or cost-minimizing development. The environmentalist is trying to minimize the impact on the environment, but realizes some development is needed; thus, the environmentalist will choose the parcels to be developed according to this guiding principle. The conservationist takes more of a "not in my back yard" (NIMBY) approach to protect selected aspects of the environment. Lastly, the land developer is driven by the profit incentive and will select those parcels that achieve the greatest financial benefit. The multiobjective model will optimize each of these broad objectives simultaneously while also satisfying certain system constraints.

We have selected just one objective per stakeholder and have chosen linear objectives with the exception of the government planner whose objective (compactness) can be modeled as a convex, quadratic function. The resulting weighted problems of form (2), which we use to generate a representative subset of Pareto solutions, are convex, quadratic mixed integer programs. In many cases, these problems are tractable and solutions can be obtained in minutes. With this approach, the level of detail benefits the decision-makers involved in planning models because they can quantify actual tradeoffs, ordinarily very hard to measure, and can therefore make more informed decisions. Essentially, the role of operations researchers/management scientists in planning is to abstract very detailed considerations into something retaining a sufficient level of detail, yet is tractable. Our level of detail retains a certain degree of realism important for analytical reasons, while affording reasonable computational times, necessary for making our approach usable as a framework for the Smart Growth initiative. Additionally, enumeration of the entire Pareto set is not needed since a representative subset conveys the important tradeoffs.

2.2. The government planner

The government planner has several goals in land allocation consistent with Smart Growth. First, the planner is interested in developing key priority funding areas. These sections have been targeted by the state to promote redevelopment of blighted urban areas and maximize existing capacity for facilities (e.g., water, sewer). Second, the planner is interested in minimizing the low density zone land parcels to minimize sprawl. Third, the planner prefers to keep the land that is developed in as compact an area as possible. In this work only the compactness objective is considered.

2.2.1. Compactness measure

There have been several mathematical approaches to minimizing the spread of development or maximizing the compactness of the development area, e.g., [19,22]. In this paper, we measure the extent of the development area as the length of the diagonal of the smallest rectangle enclosing the developed parcels, subject to a particular axis orientation described in the last part of this section. The goal is then to minimize the square of this length.

To make this notion of compactness clear, first suppose that the set of parcels fits into a rectangular grid with rows and columns assigned to each parcel. This method does not imply that each of the land parcels is rectangular or even regularly shaped, just that there is a rectangular outer envelope surrounding the parcels in questions. The rows and columns can relate to longitude and latitude or some other geographical designation, as appropriate. Consider the following depiction of this scheme in Fig. 1 with 26 columns and 15 rows.

For each parcel i:

- $\text{row}_S(i)$ = the row number south of all points in parcel i and closest to the southernmost point in parcel i.
- $\text{row}_N(i)$ = the row number north of all points in parcel i and closest to the northernmost point in parcel i.
- $\text{col}_E(i)$ = the column number east of all points in parcel i and closest to the easternmost point in parcel i.
- $\text{col}_W(i)$ = the column number west of all points in parcel i and closest to the westernmost point in parcel i.

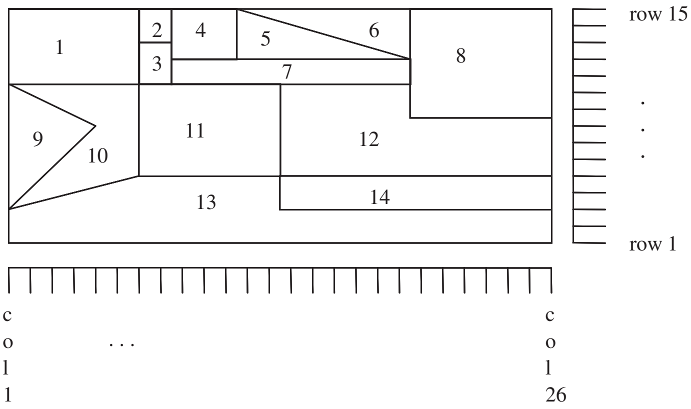

Fig. 1. Depiction of rectangular grid around parcels.

For example, for parcel 6 in Fig. 1, $\text{row}_{\text{S}}(6) = 12$, $\text{row}_{\text{N}}(6) = 15$, $\text{col}_{\text{E}}(6) = 20$, $\text{col}_{\text{W}}(6) = 12$, noting, respectively, parcel 6's southern, northern, eastern, and western borders exactly coincide with these values.

We designate the value of the variable r_{N} as the row index, which is north of all developed parcels but closest to the northernmost developed parcel. Also r_{S} refers to the row index, which is south of all developed parcels but closest to the southernmost developed parcel. Similarly, $c_{\text{E}}, c_{\text{W}}$ refer to the eastern and westernmost column indices (respectively) for this rectangle. Thus, as indicated in Fig. 2 $(r_{\text{S}}, c_{\text{W}})$ is the southwestern corner of the box, $(r_{\text{S}}, c_{\text{E}})$ is the southeastern corner, and $(r_{\text{N}}, c_{\text{W}})$, $(r_{\text{N}}, c_{\text{E}})$ are, respectively, the northwestern and northeastern corners. Formally, these relationships for $r_{\text{S}}, r_{\text{N}}, c_{\text{W}}, c_{\text{E}}$ are:

$$r_{\text{N}} = \max\{\text{row}_{\text{N}}(i) \mid d_i = 1\}, \tag{4a}$$

$$r_{\text{S}} = \min\{\text{row}_{\text{S}}(i) \mid d_i = 1\}, \tag{4b}$$

$$c_{\text{E}} = \max\{\text{col}_{\text{E}}(i) \mid d_i = 1\}, \tag{4c}$$

$$c_{\text{W}} = \min\{\text{col}_{\text{W}}(i) \mid d_i = 1\}. \tag{4d}$$

These relationships can be encoded in terms of linear constraints:

$$r_{\text{S}} - \text{row}_{\text{S}}(i) \leqslant (1 - d_i)M, \tag{5a}$$

$$\text{row}_{\text{N}}(i) - r_{\text{N}} \leqslant (1 - d_i)M, \tag{5b}$$

$$c_{\text{W}} - \text{col}_{\text{W}}(i) \leqslant (1 - d_i)M, \tag{5c}$$

$$\text{col}_{\text{E}}(i) - c_{\text{E}} \leqslant (1 - d_i)M, \tag{5d}$$

where M is a suitably large positive constant. In each developed parcel r_{S} represents the southernmost row index (or just below it) since $d_i = 1 \Rightarrow r_{\text{S}} \leqslant \text{row}_{\text{S}}(i)$. When the parcel is undeveloped, we have $d_i = 0 \Rightarrow r_{\text{S}} \leqslant \text{row}_{\text{S}}(i) + M$, which, based on the value of M, provides no

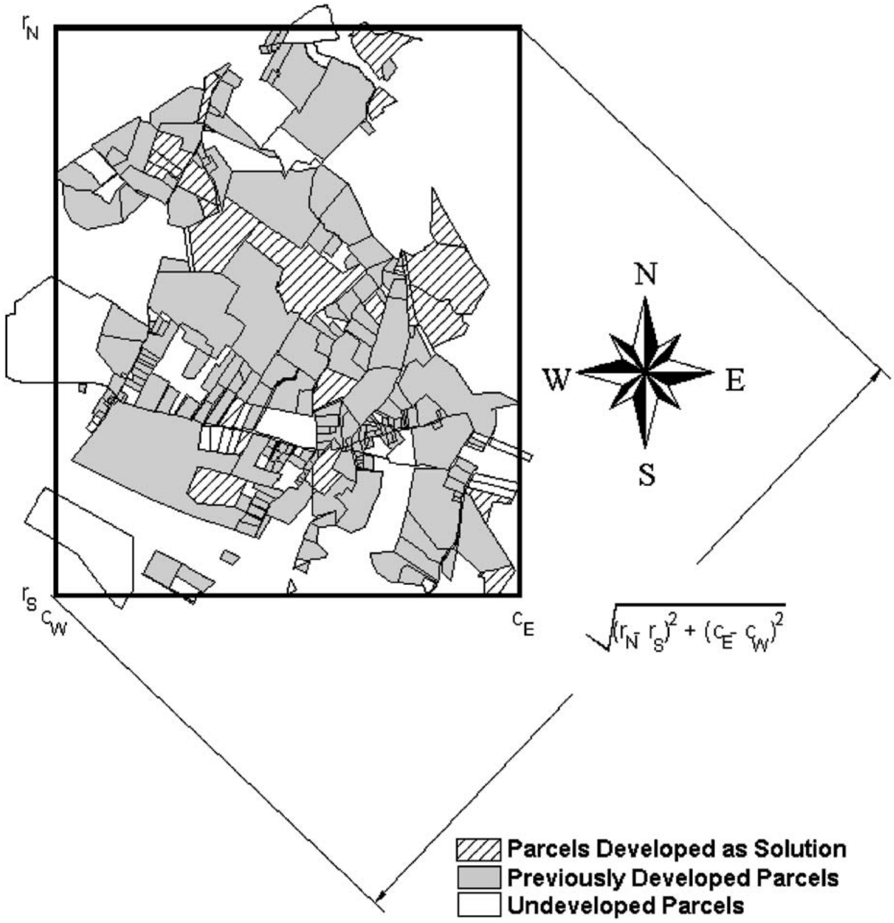

Fig. 2. Depiction of the diagonal of the outer rectangle.

restriction on r_S. We want equality holding for at least one index i for a developed parcel in (5a), a natural consequence as shown in Section 3; the logic for the other three variables follows similarly. Realistic row-column bounds on the variables are:

$$0 \leqslant r_S, r_N, c_W, c_E. \tag{5e}$$

When considering a database of both undeveloped parcels and already developed ones, we do not actually need the variables d_i for each of the previously developed parcels. All such variables would have a value of one and could simply be incorporated into (5a)–(5d). For example, (5a) reduces to $r_S - \text{row}_S(i) \leqslant 0$ if parcel i is already developed. When a large number of parcels are already developed (as was the case in our database), this step represents a huge savings in the number of binary variables needed and makes the computations more reasonable.

The length of the diagonal of the rectangle containing all the parcels selected for development is given by

$$\max\_\text{dist} = \sqrt{(r_N - r_S)^2 + (c_E - c_W)^2}. \tag{6}$$

Without loss of generality, the squared distance

$$\min\ \max\_\text{dist\_sq} = (r_N - r_S)^2 + (c_E - c_W)^2 \tag{7}$$

can be minimized.

2.2.2. Other considerations concerning compactness

Besides the compactness measurement in (7), the L_1 distance, i.e.,

$$\min\ L_1\_\text{dist} = |r_N - r_S| + |c_E - c_W| = (r_N - r_S) + (c_E - c_W) \tag{8}$$

can also be used in light of the fact that $r_N \geqslant r_S$ and $c_E \geqslant c_W$. While use of (7) results in a convex, quadratic mixed integer program as the weighting method subproblem, use of (8) creates a mixed integer linear program. The preference of (7) over (8) is because of the size of the developed area via the diagonal of the outer rectangle as compared to the sum of the two sides, i.e., half the perimeter when (8) is used. Also, employing the diagonal relates to the maximum distance that infrastructure (e.g., roads, pipes, power lines) would need to be installed. Minimizing this distance would be advantageous from the planner's point of view.

An alternative compactness objective, such as minimizing the area of the outer rectangle, i.e., $(r_N - r_S)(c_E - c_W)$, while initially attractive, can be nonconvex, and thus inappropriate for use with standard multiobjective solution methods such as the weighting method. Consequently, (7) has computational advantages over these alternative formulations as well.

Fig. 3. (a) (Left) Compactness measured with North–South axis. (b) (Right) Compactness measured with axes rotated 40°.

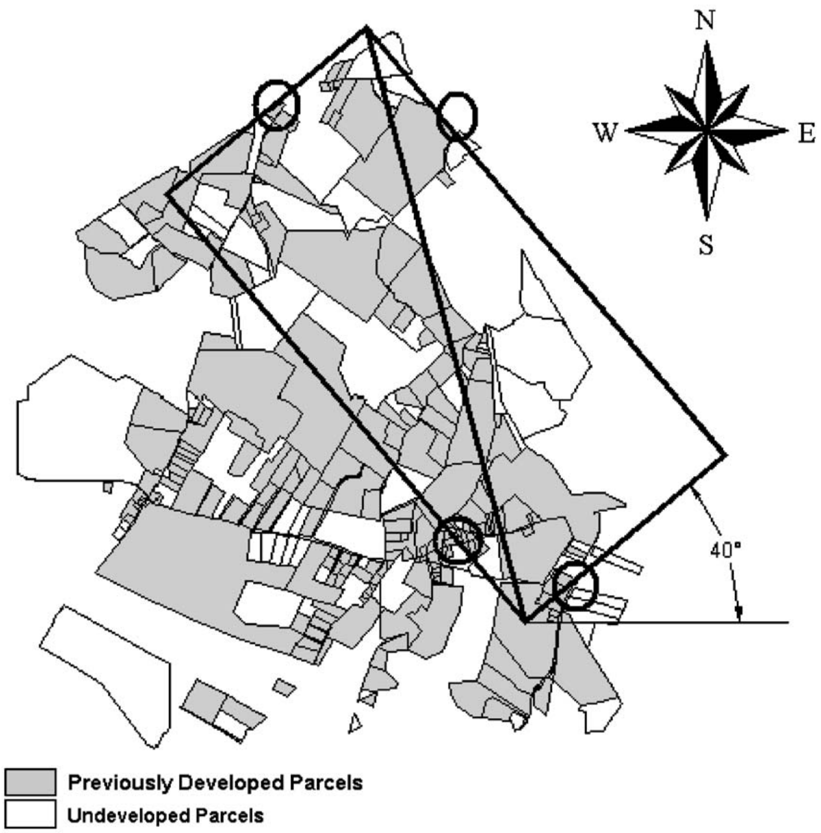

Fig. 4. Key parcels define corridor with axes rotated.

The results of the optimization depend on the orientation of the axes when measuring compactness. Consider the case presented in Fig. 3. Fig. 3a presents a subset of the parcels selected without rotating the axes, and Fig. 3b presents the same set with the axes rotated 40°.

Fig. 3b presents a rectangle that is taller and wider, resulting in a measure of compactness larger than in Fig. 3a. The extreme points that define the outer rectangle account for this difference. For another angle, the outer rectangle might be smaller.

However, this orientation-dependent aspect of the compactness measure can be used to the planner's advantage. Consider the case where the planner is interested in guiding development along a northwest–southeast corridor. By selecting a certain subset of the developed parcels (typically only a few, key ones must be chosen by the planner), along with a suitable change in axis rotation, the compactness rectangle determined by (5a)–(5d) can be shifted, as in Fig. 3b. The result is a preference for developed parcels along this designated corridor. For example, consider how Fig. 4 has circled four key parcels and a 40° shift of the axes to promote compact development along the northwest–southeast corridor.

In a slightly modified notion of compactness, we suppose that the area in question is divided into Q subdivisions where a typical one is indexed by $q \in \{1, 2, \ldots, Q\}$. The particular application will determine the appropriate number of subdivisions since the planner may want to promote a compactness measure separately for each one.[1] This choice is important if there are already developed parcels in the corners of the overall land area to be considered. To circumvent this problem, one can subdivide the area to allow for compactness to be determined within the separate subdivisions. The resulting modified notion of compactness is as follows. First, we let $\max\_\mathrm{dist\_sq}_q = (r_{N,q} - r_{S,q})^2 + (c_{E,q} - c_{W,q})^2$ with $r_{N,q}, r_{S,q}, c_{W,q}, c_{E,q}$ defined analogously as in (1) but specific to subdivision q. Then, the following constraints are enforced:

$$r_{S,q} - \mathrm{rows_S}(i) \leqslant (1 - d_i)M, \quad \text{for all } i \text{ in subdivision } q, \tag{9a}$$

$$\mathrm{row_N}(i) - r_{N,q} \leqslant (1 - d_i)M, \quad \text{for all } i \text{ in subdivision } q, \tag{9b}$$

$$c_{W,q} - \mathrm{col_W}(i) \leqslant (1 - d_i)M, \quad \text{for all } i \text{ in subdivision } q, \tag{9c}$$

$$\mathrm{col_E}(i) - c_{E,q} \leqslant (1 - d_i)M, \quad \text{for all } i \text{ in subdivision } q, \tag{9d}$$

$$0 \leqslant r_{S,q}, r_{N,q}, c_{W,q}, c_{E,q}, \quad \text{for subdivision } q. \tag{9e}$$

Constraint (9) then replaces (5) and the objective function (7) is replaced by

$$\sum_{q=1}^{Q} \max\_\mathrm{dist\_sq}_q = \sum_{q=1}^{Q} (r_{N,q} - r_{S,q})^2 + (c_{E,q} - c_{W,q})^2 \tag{10}$$

which is to be minimized. Incorporating subdivisions into the model is mostly a tool for the planner. On the computational side, allowing for subdivisions generates both more variables and more constraints, resulting in a more computationally challenging problem.

2.3. The environmentalist

The environmentalist has several objectives that can be optimized: maximizing the distance of developed land to streams to lessen the environmental impacts, concentrating development in hydrological unit codes (HUCs) that already have had substantial development, and minimizing the global change in imperviousness of the land development. We concentrate on the imperviousness measure, identified by past researchers (e.g. [28,29]), as an effective index of urban impact. As the level of imperviousness increases, the streams where the impervious area drains experience negative impacts, such as increased high flows, decreased base flows, thermal shocks, and greater nutrient and pollutant loads [29–31]. Since the US Geological Survey's National Land Cover Database (NLCD), provides a spatially explicit account of imperviousness across the country at a resolution of 30 m [32], imperviousness has now become a readily measured quantity. Consequently, the following objective is used for the environmentalist:

$$\min \sum_{i \in S} \Delta\_\mathrm{imperv}_i \mathrm{area}_i d_i, \tag{11}$$

where Δ_imperv_i is the change in imperviousness if the parcel is developed and area_i is the total area of the parcel.

[1]This strategy may also include separate axis orientation for each subdivision.

2.4. The conservationist

The conservationist occupies the most environmentally friendly position along the spectrum of interests of the four stakeholders. This stakeholder is adamant about protecting certain key parcels denoted by the set $\tilde{S}$ from development. In terms of an objective function, this need leads to:

$$\min \sum_{i \in \tilde{S}} \text{area}_i \, d_i, \tag{12}$$

i.e., minimize the total area of environmentally sensitive parcels to be developed to protect the flora and fauna in these areas. This stance is extreme since the conservationist's objective is optimized if absolutely no development takes place within the key parcels identified by this stakeholder. Examples of this stance persistently appear in the media when individuals try to halt planned development in locations they wish to protect. A current example in Montgomery County, Maryland, is the Eyes of the Paint Branch, a watershed association opposed to the planned construction of a major highway that will encroach on a stream of naturally reproducing trout.

2.5. The land developer

The developer is modeled to maximize the total value of the developed parcels where the value is calculated for a parcel i:[2]

$$\text{value}_i = 0.20 \begin{cases} \text{avg\_sales}_{\text{LD}} * \left(\dfrac{\text{area}_i}{\text{density}_{\text{LD}}}\right) \text{if parcel } i \text{ is low density, residential,} \\ \text{avg\_sales}_{\text{MD}} * \left(\dfrac{\text{area}_i}{\text{density}_{\text{MD}}}\right) \text{if parcel } i \text{ is medium density, residential,} \\ \text{avg\_sales}_{\text{HD}} * \left(\dfrac{\text{area}_i}{\text{density}_{\text{HD}}}\right) \text{if parcel } i \text{ is high density, residential,} \\ \text{avg\_sales\_sq\_area}_{\text{COM}} * (a_{\text{COM}} + b_{\text{COM}} \text{area}_i) \text{ if parcel } i \text{ is commercial,} \\ \text{avg\_sales\_sq\_area}_{\text{IND}} * (a_{\text{IND}} + b_{\text{IND}} \text{area}_i) \text{ if parcel } i \text{ is industrial,} \end{cases} \tag{13}$$

where

- density is the number of acres per unit (e.g., 1 acre/unit for residential low density);[3]
- value_i is the value of parcel i if developed (US\$) consistent with industry standards; we take 80% of this value as costs so the net value is 20% of the right-hand side of (13) enclosed in the braces; we have assumed that this 80% cost is already taken out in what follows;
- $\text{avg\_sales}_{\text{LD}}$, $\text{avg\_sales}_{\text{MD}}$, $\text{avg\_sales}_{\text{HD}}$ is the average sales dollars/unit for low density, medium density, and high density residential parcels taken in the recent years, respectively (see Table 1);

[2]The average sales per square area value (avg_sales_sq_area) used square feet is the square area in question given the original form of the data with 1 square foot equal to $0.0929\,\text{m}^2$.

[3]In this sense, density is the inverse of what is sometimes used, e.g., units/acre, 1 acre equals 0.405 h.

Table 1
Average sales by residential zone

$\text{avg\_sales}_{\text{LD}}$	$\text{avg\_sales}_{\text{MD}}$	$\text{avg\_sales}_{\text{HD}}$
\$449,540	\$291,366	\$256,658

- $\frac{\text{area}_i}{\text{density}_{\text{LD}}}, \frac{\text{area}_i}{\text{density}_{\text{MD}}}, \frac{\text{area}_i}{\text{density}_{\text{HD}}}$ are the estimates for the maximum number of units possible on the parcel if it is a low density, medium density, or high-density residential parcel, respectively;
- $\text{avg\_sales\_sq\_area}_{\text{COM}}$, $\text{avg\_sales\_sq\_area\_}_{\text{IND}}$ are the average ratio of sales dollars for a unit to square area of the structure for commercial and industrial parcels, respectively;
- $a_{\text{COM}} + b_{\text{COM}}\text{area}_i$, $a_{\text{IND}} + b_{\text{IND}}\text{area}_i$ are statistically estimated relationships between the area of the parcel and the square area for commercial and industrial parcels, respectively, useful for predicting the typical area of structures on yet undeveloped parcels.

Parcels are grouped into the following sets:

$$\text{if zoning code}_i = \begin{cases} \text{“11” then parcel } i \text{ is designated as a 1 acre low-density residential lot,} \\ \quad \text{the set of parcels is } S_{11}, \\ \text{“12” then parcel } i \text{ is designated as a } 1/4 \text{ acre medium density} \\ \quad \text{residential lot, the set of parcels is } S_{12}, \\ \text{“13” then parcel } i \text{ is designated as a } 1/8 \text{ acre high-density residential} \\ \quad \text{lot, the set of parcels is } S_{13}, \\ \text{“14” then parcel } i \text{ is designated as a commerical lot, the set of} \\ \quad \text{parcels is } S_{14}, \\ \text{“15” then parcel } i \text{ is designated as an industrial lot, the set of} \\ \quad \text{parcels is } S_{15}, \text{ otherwise then parcel } i\text{'s designation is} \\ \quad \text{unassigned and is to be decided upon by the model,} \\ \quad \text{the set of parcels is } S_{99}. \end{cases} \tag{14}$$

For each parcel $i \in S_{99}$, hereafter called an unassigned parcel, the following constraint is needed:

$$d_i = \text{RLD}_i + \text{RMD}_i + \text{RHD}_i + \text{COM}_i + \text{IND}_i \quad \text{for all } i \in S_{99}, \tag{15}$$

where $\text{RLD}_i, \text{RMD}_i, \text{RHD}_i, \text{COM}_i, \text{IND}_i \in \{0, 1\}$ for all $i \in S_{99}$.

These variables represent whether the unassigned parcel is selected to be residential low density, residential medium density, residential high density, commercial, or industrial, with exactly one of these choices made if the parcel is developed. Consequently, the objective function for the developer becomes:

$$\max \sum_{i\in S_{11}} \text{value}_i\, d_i + \sum_{i\in S_{12}} \text{value}_i\, d_i + \sum_{i\in S_{13}} \text{value}_i\, d_i + \sum_{i\in S_{14}} \text{value}_i\, d_i + \sum_{i\in S_{15}} \text{value}_i\, d_i$$
$$+ \sum_{i\in S_{99}} (\text{value}_i\, \text{RLD}_i + \text{value}_i\, \text{RMD}_i + \text{value}_i\, \text{RHD}_i + \text{value}_i\, \text{COM}_i + \text{value}_i\, \text{IND}_i). \quad (16)$$

2.6. Additional constraints

2.6.1. Growth rates on number of units and acres by zone

Based on 5-year projections for growth rates of number of housing units for residential areas and hectares (acres) for commercial and industrial sites, constraints that provide lower and upper bounds for these target values are included. The lower and upper bounds represent, respectively, −20% and +20% of these rates. We note that each of these designations takes parcels from a fixed set (i.e., for RLD it's 11), as well as from the set of unassigned parcels (i.e., code equal to 99). Consequently, realistic bounds on new development for the residential low density parcels are:

$$\min_{\text{RLD}} \leqslant \sum_{i\in S_{11}} \text{units}_i\, d_i + \sum_{i\in S_{99}} \text{units}_i\, \text{RLD}_i \leqslant \max_{\text{RLD}}, \quad (17a)$$

where $\min_{\text{RLD}}$ and $\max_{\text{RLD}}$ represent, respectively, the minimum and maximum number of units to be developed, and units_i represents the positive number of units that can be developed for parcel i. Similar constraints for the other four zones are:

$$\min_{\text{RMD}} \leqslant \sum_{i\in S_{12}} \text{units}_i\, d_i + \sum_{i\in S_{99}} \text{units}_i\, \text{RMD}_i \leqslant \max_{\text{RMD}}, \quad (17b)$$

$$\min_{\text{RHD}} \leqslant \sum_{i\in S_{13}} \text{units}_i\, d_i + \sum_{i\in S_{99}} \text{units}_i\, \text{RHD}_i \leqslant \max_{\text{RHD}}, \quad (17c)$$

$$\min_{\text{COM}} \leqslant \sum_{i\in S_{14}} \text{acres}_i\, d_i + \sum_{i\in S_{99}} \text{acres}_i\, \text{COM}_i \leqslant \max_{\text{COM}}, \quad (17d)$$

$$\min_{\text{IND}} \leqslant \sum_{i\in S_{15}} \text{acres}_i\, d_i + \sum_{i\in S_{99}} \text{acres}_i\, \text{IND}_i \leqslant \max_{\text{IND}}, \quad (17e)$$

where the minimum and maximum parameters are defined analogously, except for the commercial and industrial parcels, where these values refer to hectares (acres) based on the number of acres acres_i for the parcel.

2.6.2. First develop assigned parcels for a zone

Another set of constraints involving the classification of the unassigned parcels S_{99} ensures that these parcels are not developed when there are sufficient units in the existing pool of parcels. The rationale is the bureaucratic effort needed to subdivide and rezone large essentially unzoned land, i.e., rural density transfer, represents a significant impediment to development. Consequently, all undeveloped but acceptably zoned parcels will undergo development first. This logic is consistent with minimizing the bureaucratic burden of establishing zones for unassigned parcels. These restrictions can be enforced with the constraints:

$$\sum_{i \in S_{99}} \mathrm{RLD}_i\, \mathrm{units}_i \leqslant M y_{\mathrm{RLD}}, \quad \sum_{i \in S_{11}} \mathrm{units}_i - \min\nolimits_{\mathrm{RLD}} \leqslant M(1 - y_{\mathrm{RLD}}), \quad y_{\mathrm{RLD}} \in \{0, 1\}, \tag{18a}$$

where M is a suitably large positive number and the residential low density zone is represented.[4]

We see that, for example, if $\sum_{i \in S_{11}} \mathrm{units}_i > \min_{\mathrm{RLD}}$ with enough parcels in the assigned pool for residential low density, then necessarily $y_{\mathrm{RLD}} = 0$, which forces $\mathrm{RLD}_i = 0$, for each $i \in S_{99}$, or that no units from unassigned parcels get converted to residential low density and none are developed. Conversely, if $\sum_{i \in S_{11}} \mathrm{units}_i \leqslant \min_{\mathrm{RLD}}$, then the binary variable y_{RLD} can have a value of either 0 or 1. When a value of 1 is chosen, since M was chosen sufficiently large, no restrictions are posed on the potential residential low-density parcels coming from the unassigned group. Otherwise, when a value of 0 is selected, none of these other parcels are converted to residential low density. The former case, all things being equal, will be selected when $\sum_{i \in S_{11}} \mathrm{units}_i \leqslant \min_{\mathrm{RLD}}$, since it allows for a large, feasible region, and hence a better objective function value. Similar reasoning holds for the other four zones based on these four constraints:

$$\sum_{i \in S_{99}} \mathrm{RMD}_i\, \mathrm{units}_i \leqslant M y_{\mathrm{RMD}}, \quad \sum_{i \in S_{12}} \mathrm{units}_i - \min\nolimits_{\mathrm{RMD}} \leqslant M(1 - y_{\mathrm{RMD}}), \quad y_{\mathrm{RMD}} \in \{0, 1\}, \tag{18b}$$

$$\sum_{i \in S_{99}} \mathrm{RHD}_i\, \mathrm{units}_i \leqslant M y_{\mathrm{RHD}}, \quad \sum_{i \in S_{13}} \mathrm{units}_i - \min\nolimits_{\mathrm{RHD}} \leqslant M(1 - y_{\mathrm{RHD}}), \quad y_{\mathrm{RHD}} \in \{0, 1\}, \tag{18c}$$

$$\sum_{i \in S_{99}} \mathrm{COM}_i\, \mathrm{acres}_i \leqslant M y_{\mathrm{COM}}, \quad \sum_{i \in S_{14}} \mathrm{acres}_i - \min\nolimits_{\mathrm{COM}} \leqslant M(1 - y_{\mathrm{COM}}), \quad y_{\mathrm{COM}} \in \{0, 1\}, \tag{18d}$$

$$\sum_{i \in S_{99}} \mathrm{IND}_i\, \mathrm{acres}_i \leqslant M y_{\mathrm{IND}}, \quad \sum_{i \in S_{15}} \mathrm{acres}_i - \min\nolimits_{\mathrm{IND}} \leqslant M(1 - y_{\mathrm{IND}}), \quad y_{\mathrm{IND}} \in \{0, 1\}. \tag{18e}$$

The next set of constraints involving the unassigned parcels ensures that all the assigned ones, i.e., those in $S_{11}, S_{12}, S_{13}, S_{14}, S_{15}$, are used completely if there is an insufficient number relative to the lower bounds in (17). The rationale for these constraints is similar to the logic in the previous section. For the constraints below, N is a suitably large positive number, and using the residential low-density parcels as an example results in

$$\min\nolimits_{\mathrm{RLD}} - \sum_{i \in S_{11}} \mathrm{units}_i \leqslant N(1 - w_{\mathrm{RLD}}), \quad \sum_{i \in S_{11}} \mathrm{units}_i - \sum_{i \in S_{11}} \mathrm{units}_i\, d_i \leqslant N w_{\mathrm{RLD}}, \quad w_{\mathrm{RLD}} \in \{0, 1\}. \tag{19a}$$

This constraint shows, for example, that if the existing residential low-density units S_{11} are insufficient to meet even the minimum growth goal of $\min_{\mathrm{RLD}}$, that is if $\min_{\mathrm{RLD}} > \sum_{i \in S_{11}} \mathrm{units}_i$, then the binary variable w_{RLD} must equal 0. This result in turn implies $\sum_{i \in S_{11}} \mathrm{units}_i \leqslant \sum_{i \in S_{11}} \mathrm{units}_i\, d_i$. In

[4] If, in a particular implementation of our proposed model, pre-processing of the data before sending it to a solver was considered, certain speedups could be used. For example, forcing $\mathrm{RLD}_i = 0$ if $\sum_{i \in S_{11}} \mathrm{units}_i \geqslant \min_{\mathrm{RLD}}$. These and other implementation-specific details have been omitted in order to present the most general form of the model without resorting to pre-processing of any data.

combination with the fact that the inequality $\sum_{i\in S_{11}} \text{units}_i\, d_i \leqslant \sum_{i\in S_{11}} \text{units}_i$ is always true, the desired result of $\sum_{i\in S_{11}} \text{units}_i\, d_i = \sum_{i\in S_{11}} \text{units}_i$ or that $d_i = 1,\ \forall i \in S_{11}$ follows since there is always a positive number of units on each parcel. Similar logic applies to the other four zones. Conversely, when there is a sufficient number of residential low-density units, i.e., $\min_{\text{RLD}} \leqslant \sum_{i\in S_{11}} \text{units}_i$, w_{RLD} can equal either 0 or 1. A value of 0 will force $d_i = 1, \forall i \in S_{11}$ since $\sum_{i\in S_{11}} \text{units}_i \leqslant \sum_{i\in S_{11}} \text{units}_i\, d_i$, a value of w_{RLD} equal to 1 will place no constraints on these units. All things being equal, a value of w_{RLD} equal to 1 will be preferred by the model since it allows for a larger feasible region. Similar variables and constraints also hold for the other four zones, resulting in these constraints:

$$\min_{\text{RMD}} - \sum_{i\in S_{12}} \text{units}_i \leqslant N(1 - w_{\text{RMD}}), \quad \sum_{i\in S_{12}} \text{units}_i - \sum_{i\in S_{12}} \text{units}_i\, d_i \leqslant N w_{\text{RMD}}, \quad w_{\text{RMD}} \in \{0, 1\}, \tag{19b}$$

$$\min_{\text{RHD}} - \sum_{i\in S_{13}} \text{units}_i \leqslant N(1 - w_{\text{RHD}}), \quad \sum_{i\in S_{13}} \text{units}_i - \sum_{i\in S_{13}} \text{units}_i\, d_i \leqslant N w_{\text{RHD}}, \quad w_{\text{RHD}} \in \{0, 1\}, \tag{19c}$$

$$\min_{\text{COM}} - \sum_{i\in S_{14}} \text{acres}_i \leqslant N(1 - w_{\text{COM}}), \quad \sum_{i\in S_{14}} \text{acres}_i - \sum_{i\in S_{14}} \text{acres}_i\, d_i \leqslant N w_{\text{COM}}, \quad w_{\text{COM}} \in \{0, 1\}, \tag{19d}$$

$$\min_{\text{IND}} - \sum_{i\in S_{15}} \text{acres}_i \leqslant N(1 - w_{\text{IND}}), \quad \sum_{i\in S_{15}} \text{acres}_i - \sum_{i\in S_{15}} \text{acres}_i\, d_i \leqslant N w_{\text{IND}}, \quad w_{\text{IND}} \in \{0, 1\}. \tag{19e}$$

2.7. The multiobjective optimization model

The resulting multiobjective optimization problem for Smart Growth is:

$$\begin{aligned}
\min z_1 &= \sum_{q=1}^{Q} (r_{\text{N},q} - r_{\text{S},q})^2 + (c_{\text{E},q} - c_{\text{W},q})^2 \text{ (planner)},\\
\min z_2 &= \sum_{i\in S} (\Delta_\text{imperv}_i)(\text{area}_i) d_i \text{ (environmentalist)},\\
\min z_3 &= \sum_{i\in \bar{S}} \text{area}_i\, d_i, \text{ (conservationist)},\\
\max z_4 &= \sum_{i\in S_{11}} \text{value}_i\, d_i + \sum_{i\in S_{12}} \text{value}_i\, d_i + \sum_{i\in S_{13}} \text{value}_i\, d_i + \sum_{i\in S_{14}} \text{value}_i\, d_i + \sum_{i\in S_{15}} \text{value}_i\, d_i\\
&\quad + \sum_{i\in S_{99}} (\text{value}_i\, \text{RLD}_i + \text{value}_i\, \text{RMD}_i + \text{value}_i\, \text{RHD}_i + \text{value}_i\, \text{COM}_i + \text{value}_i\, \text{IND}_i)\\
&\quad \text{(developer)}
\end{aligned}$$

$$\begin{aligned}
&\text{s.t.}\\
&(9), (15), (17)-(19)\\
&d_i \in \{0, 1\}, \quad \forall i \in S\\
&\mathrm{RLD}_i, \mathrm{RMD}_i, \mathrm{RHD}_i, \mathrm{COM}_i, \mathrm{IND}_i \in \{0, 1\}, \quad \forall i \in S_{99}.
\end{aligned} \tag{20}$$

Pareto optimal solutions to (20) can be obtained, among other approaches, via the weighting method [24]. In the weighting method, each objective is multiplied by a positive weight and then summed to form a single objective weighted problem whose feasible region is the same as (20).[5] Different Pareto optimal points can be generated by choosing different weights. For nonconvex problems there may be duality gap points, which are Pareto optimal solutions that cannot be obtained via this method (other approaches can be used in this case); for an example of these duality gaps, see [20, p. 560]. Since we are not concerned with enumerating every Pareto optimal solution, these gap points do not pose a problem in this setting.

3. Existence results

This section presents some theoretical results concerning the existence of Pareto optimal solutions to the multiobjective optimization model (20).

Both the objective function and the constraint set of (20) are convex which is important for computational reasons. This result will ensure that all local solutions are in fact global ones [33]. Consequently, all the pieces of the weighted objective function are linear, except for the compactness measure, which is convex quadratic, resulting, as shown below, in a convex quadratic objective function overall. The constraints are linear except for the binary restrictions on selected variables. Thus, a mixed integer convex quadratic problem results, whose relaxed version is a convex quadratic program.

Theorem 1. *The weighted objective* $w_1z_1 + w_2z_2 + w_3z_3 - w_4z_4$ *is convex in its variables as long as the weights* $w_1, w_2, w_3, w_4 \geqslant 0$.

Proof. (see appendix).

We still need to ensure that (9a)–(9d) accurately define the borders of the rectangle around the developed parcels. This result is shown in the next theorem.

Theorem 2. *Assuming the weighted problem to* (20) *has an optimal solution, and if Assumption* 1 *holds, then constraints* (9a)–(9d) *ensure that* $r_{N,q}, r_{S,q}, c_{E,q}, c_{W,q}$ *correspond respectively to the northernmost, southernmost, easternmost, and westernmost borders of all the developed parcels in subdivision* q *for* $q = 1, \ldots, Q$.

[5]The developer's objective which is to be maximized is first multiplied by -1 to convert it to an appropriate minimization, consistent with the other objectives.

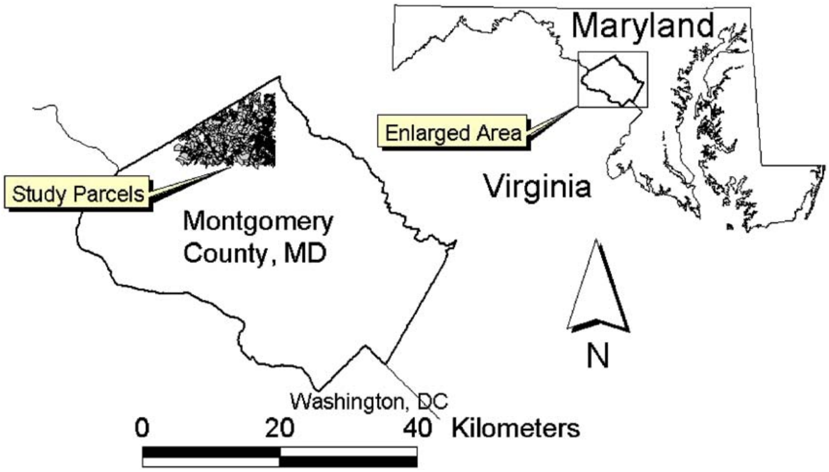

Fig. 5. Montgomery County, Maryland.

Proof. (see appendix).

The main result shown in Theorem 3 then follows as an immediate consequence of the weighting method, assuming that at least one subproblem of form (2) has a solution.

Theorem 3. *The Smart Growth problem* (20) *always has a Pareto optimal solution.*

4. Numerical results for Montgomery County, Maryland

This section presents numerical results based on solving the multiobjective optimization problem (20) for land parcels in Montgomery County, Maryland. Pareto optimal solutions to (20) can be obtained as solutions to the weighted version of the problem, which are instances of QMIPs with about 3500 variables (most of which are binary) and over 23,000 constraints.

4.1. Database of land parcels for Montgomery County, Maryland

Montgomery County, Maryland, is located north of Washington, DC and borders the state of Virginia, as shown in Fig. 5. Covering some 1300 km^2 (500 $miles^2$) of Maryland and occupied by over 873,000 inhabitants,[6] this county is the most populous in Maryland. Using a database of Montgomery County land parcel information in GIS format, both current and potential development of the area were analyzed. Fig. 6 shows the northwestern section of the county used in this study, comprising our database of some 913 undeveloped and 4837 previously developed parcels.

[6]According to the 2000 Census survey; source: http://www.co.mo.md.us/cntymap.htm

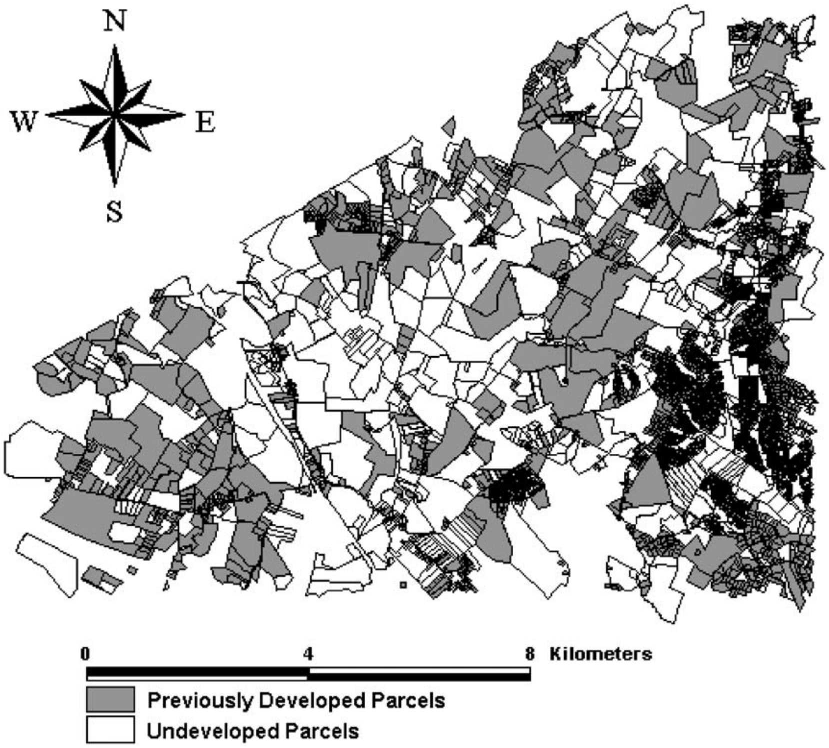

Fig. 6. Montgomery County, Maryland database segmented by previously developed parcels (gray) and those undeveloped yet available for development (white).

For the purposes of examining the compactness objectives, the county was divided into four subdivisions or quadrants (Q1, Q2, Q3, Q4) as shown in Fig. 7a. Since the borders of the parcels were not perfectly aligned with the quadrant divisions, the centroid of each parcel was used to determine into which quadrant the parcel should be assigned. If the centroid was within the bounds of the quadrant, then the whole parcel was assigned to that quadrant. We note that quadrant 3 (Q3), all things being equal, had the greatest chance for significant compact land development given its relatively small number of previously developed parcels. After partitioning the parcels based on this centroid rule, the resulting quadrants and their associated parcels appear in Fig. 7b. Once the parcels were assigned to the quadrants, the parcel coordinates were normalized for more balanced results in the weighted optimizations. Specifically, the minimum northing (row) and easting (column) values among all parcels were deducted, respectively, from the northing and easting coordinates for each parcel. Thus, the westernmost point of the westernmost parcel of the set had a horizontal coordinate of zero; similarly, the southernmost point of the southernmost parcel had a vertical coordinate of zero.

Based on our data set of residential parcels, we estimated the following parameters per unit.

The densities of the residential areas, consistent with definitions used by both the Maryland Department of Planning [34] and the Natural Resources Conservation Service [35], were taken as follows where "du" means dwelling unit, "ha" is hectare", and "ac" is acre (Table 2).

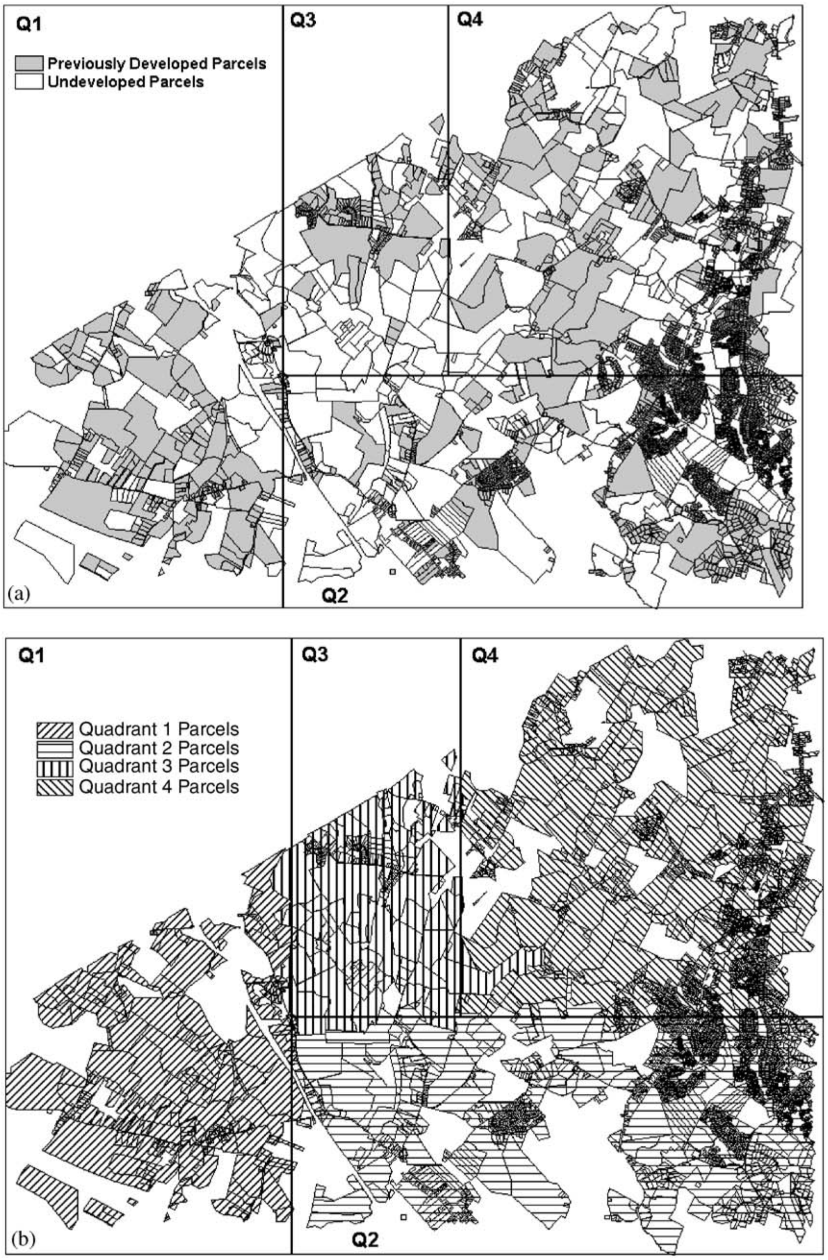

Fig. 7. (a) (Top) Division of Montgomery County study. (b) (Bottom) Parcels assigned to each quadrant.

Based on our data set of commercial and industrial parcels, we estimated the following parameters (Table 3):

To illustrate the effect of the environmentally sensitive parcels involved in the conservationist's objective function, we selected 70 parcels from our database. Their locations are shown in Fig. 8, along with the relative number in each of the quadrants as indicated in Table 4.

Table 2
Land densities by residential zone

Low density	Medium density	High density
2.47 du/ha (1 du/ac)	9.88 du/ha (4 du/ac)	19.8 du/ha (8 du/ac)

Table 3
Commercial and industrial estimated parameters

Zoning category	avg_sales_sq_area	*a*	*b*
Commercial	315.6	15,553	9736.9
Industrial	192.8	9242.2	11,604

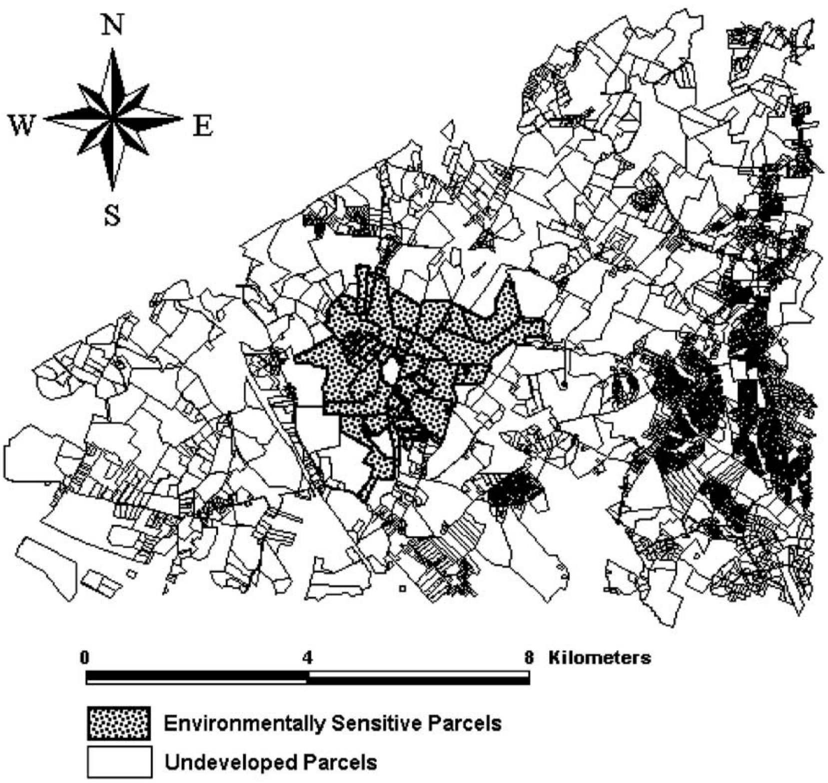

Fig. 8. Set of environmentally sensitive parcels.

4.2. Nine cases

The current and subsequent sections describe findings associated with Pareto optimal land development solutions to (20) using the weighting method. The resulting QMIPs were generated

Table 4
Number of environmentally sensitive parcels distribution by quadrant

Quadrant	Number of parcels
1	0
2	32
3	35
4	3
Total	70

Table 5
Weights assigned to each stakeholder's objective

Case		Planner (compactness)	Environmentalist (imperviousness change)	Conservationist (env. sensitive area)	Developer (profit)	Relative gap
1	Planner alone	1	0	0	0	5e−005
2	Planner Pareto	1	0.001	0.001	0.001	5e−005
3	Environmentalist alone	0	1	0	0	5e−005
4	Environmentalist Pareto	0.001	1	0.001	0.001	5e−005
5	Conservationist alone	0	0	1	0	5e−005
6	Conservationist Pareto	0.001	0.001	1	0.001	5e−005
7	Developer alone	0	0	0	1	5e−005
8	Developer Pareto[a]	0.001	0.001	0.001	1	5e−004
9	All perspectives	1	1	1	1	5e−005

[a]A relative gap of 5e−005 was not achievable within a reasonable amount of time. We thus slightly relaxed the problem and it solved with a relative gap of 5e−004.

making use of the MPL modeling language and solved using the XPRESS-MP solver. In all, nine sets of weights for the four stakeholders were applied; these weights appear in Table 5 and are displayed in 4-tuples of the form (w_1, w_2, w_3, w_4) corresponding, respectively, to the planner, the environmentalist, the conservationist, and the developer. For example, case 9 represents a weight of 1 for each of the stakeholders.

These nine cases require different objective weights for the weighted problem; there are two main groups of cases. Group one is from the single perspective of one stakeholder. For example, case 1 has weights of (1,0,0,0), which correspond to considering only the planner's perspective. Consequently, there are four of these single objective optimization cases: case 1 (‘‘Planner Alone’’), case 3 (‘‘Environmentalist Alone’’), case 5 (‘‘Conservationist Alone’’), and case 7 (‘‘Developer Alone’’). Land development plans determined as solutions to these four optimizations do not necessarily represent Pareto optimal solutions (unless they are unique solutions [11]). These results are for purposes of comparison with the Pareto optimal solutions.

The second group of cases considers strictly positive weights for each of the stakeholder perspectives resulting in Pareto optimal solutions (indicated by ‘‘Pareto’’ in the title of these cases). Case 9 involves an equal weight of 1 for each of the stakeholder's objectives. This case is contrasted with the other four cases (cases 2, 4, 6, and 8) in which one of the stakeholders is highlighted with the largest weight of one assigned to it; the weights for the other three stakeholders is set to 0.001. For example, case 2 assigns a weight of one to the planner and 0.001 to the other three stakeholders. In addition, the last column of this table represents the relative gap value, i.e., $|\text{best solution} - \text{best bound}|/\text{best bound}$, used with the solver, a value of zero generally not leading to reasonable solution times. We tried to use the same relative gap (5e−005) for all the cases, but found that the time to find the solution for the Developer Pareto case lengthened dramatically. Thus, we increased the relative gap from 5e−005 to 5e−004 for this troublesome case to save time without sacrificing solution quality. With these relative gap values selected, the range of computational times were from less than a second (0.29 s) for the Conservationist Alone case to a little over 6 h (6 h and 9 min) for the Developer Pareto case. Table 6 presents the values of the different objectives evaluated for each of these nine cases.

Analysis of these nine cases focuses on two areas. First, we examine the tradeoffs between the various stakeholders for Pareto optimal vs. single objective solutions. Next, we focus on the planner's compactness objective and highlight some key findings.

For the analysts using our model, the selection of the weights is not a trivial task since different weights will generally produce different results [27]. The selection of the weights depends on the importance of each objective, the context of the problem, and the scaling factors [27]. Miettinen [11], however, points out the confusion in whether the weights reflect the importance of each objective or the rate at which the decision maker is willing to trade off values of the objective functions. Previous analysis [36] based on fixing the sum of the weights has been done, but is most applicable when only two objective functions are present. With two objectives, one could just compute the weighted average by setting $w_1 + w_2 = 1$ and then solving $\{\min : w_1 f_1(x) + (1 - w_1) f_2(x)\}$ subject to the feasibility constraints for different values of w_1 in the range (0,1). Another approach to set the weights is the Analytical Hierarchy Process [37]. This process is a mathematical approach to evaluate preferences among disparate stakeholders and could be used in an interactive mode, as discussed in Section 4.5.

4.3. Analysis of tradeoffs

Table 6 illustrates how the individual objectives reach their optimal values when they are evaluated alone (shown in bold) and when other stakeholder interests are considered. Thus, this table provides valuable information on the explicit tradeoffs made in considering all the stakeholder perspectives, and is therefore important in the Smart Growth planning process.[7] When considering the single objective optimization, the conservationist achieves an objective of 0 (i.e., no environmentally sensitive parcels are developed).

Consider first the perspective of the planner who is trying to maximize the compactness of the developed land in all four of the quadrants taken separately. If we consider just the planner's single objective by itself (case 1), the optimal level of compactness of the developed land[8] is

[7]Different weights may produce different tradeoffs. Table 6 is meant for illustrative purposes.

[8]As measured by minimizing the square of the length of the diagonal of the outer rectangle that surrounds all developed parcels.

Table 6
Value of the objective functions by case

Case	Description	Maximum distance squared km^2 (mi^2)	Percentage of optimal	Imperviousness change km^2 (mi^2)	Percentage of optimal
1	Planner alone	**286.06 (110.45)**	100.0%	0.01840 (0.0071)	115.5%
2	Planner Pareto	286.06 (110.45)	100.0%	0.02391 (0.00923)	150.1%
3	Environmentalist alone	318.72 (123.06)	111.4%	**0.01593 (0.00615)**	100.0%
4	Environmentalist Pareto	286.06 (110.45)	100.0%	0.01594 (0.00616)	100.1%
5	Conservationist alone	325.96 (125.86)	113.9%	0.01755 (0.00678)	110.2%
6	Conservationist Pareto	286.06 (110.45)	100.0%	0.02377 (0.00918)	149.2%
7	Developer alone	333.69 (128.84)	116.7%	0.02384 (0.0092)	149.6%
8	Developer Pareto	286.06 (110.45)	100.0%	0.02374 (0.00917)	149.1%
9	All perspectives	286.06 (110.45)	100.0%	0.02337 (0.00902)	146.7%
	Numbers are **better** if:	**Smaller**	**Smaller**	**Smaller**	**Smaller**
Case	Description	Env. sensitive area km^2 (mi^2)	Percentage of optimal	Profit millions of $ US	Percentage of optimal
1	Planner alone	0.87 (553.80)	Infinite	$1317.56	69.2%
2	Planner Pareto	2.68 (1712.49)	Infinite	$1686.95	88.7%
3	Environmentalist alone	1.86 (1192.03)	Infinite	$1148.84	60.4%
4	Environmentalist Pareto	1.30 (833.64)	Infinite	$1273.13	66.9%
5	Conservationist alone	**0.00 (0.00)**	0/0	$1266.36	66.5%
6	Conservationist Pareto	0.02 (11.67)	Infinite	$1891.61	99.4%
7	Developer alone	4.09 (2616.62)	Infinite	$**1902.89**	100.0%
8	Developer Pareto	1.77 (1132.39)	Infinite	$1899.82	99.8%
9	All perspectives	2.56 (1640.65)	Infinite	$1672.72	87.9%
	Numbers are **better** if:	**Smaller**		**Larger**	**Larger**

$286.06\,\text{km}^2$ ($110.45\,\text{miles}^2$). Normalizing so that this value is 100%, the planner does worse when the other three stakeholders' objectives are optimized one at a time. In particular, the compactness measures worsen by 11.4%, 13.9%, and 16.7%, respectively, when optimizing just for the environmentalist, the conservationist, and the developer, since other concerns are more important for these other stakeholders. However when the five Pareto cases are considered ("Planner Pareto", "Environmentalist Pareto", "Conservationist Pareto", "Developer Pareto", "All Perspectives"), the optimal compactness matches the "Planner Alone" case.

The environmentalist obtains a solution with minimum change in imperviousness when his perspective is considered by itself, resulting in an optimal value of $0.01593\,\text{km}^2$. Once the other stakeholders are considered either separately or from a Pareto perspective, the environmentalist does worse. The environmentalist's objective appears to be more sensitive than the planner's since the former worsens by abo ut 50% under the "Planner Pareto" perspective, but the planner does no worse than its single objective case under "Environmentalist Pareto". The environmentalist's objective also does particularly poorly under the "Developer Pareto" perspective. Table 6 indicates a 49.1% worsening in the change in imperviousness due to accommodating the developer's objective with a higher weight. Also, the environmentalist's objective similarly suffers when considering the "Conservationist Pareto" case. Consequently, the environmentalist appears to be the most sensitive to the objectives of the other stakeholders since it has the largest percentage deviations from optimality when considering the other stakeholders.

The conservationist is able to steer development out of the environmentally sensitive areas when it is the only perspective. However, only a slight change occurs in this objective function when the other perspectives receive a small positive weight (the "Conservationist Pareto" case). Lastly, Table 6 indicates some significant worsening in the developer's optimal objective function when the other stakeholders are involved. For example, the developer's profit drops by about 33% when considering the "Environmentalist Pareto" case.

In Table 7, the developer scenarios selected are nearly or equal to the maximum possible number of parcels without exceeding the upper bound. This conclusion makes sense because profit increases as more parcels are developed. The limiting factors are the upper bounds and other constraints or perspectives that need to be considered. In contrast, the environmentalist chose to develop nearly or equal to the minimum amounts required by the bounds, reflecting the goal of minimal increase in imperviousness.

In Fig. 9, the data for the nine cases are also presented using a Value Path graph, a widely used technique to present multiobjective solutions [11,27]. The actual numbers for this analysis appear in Table 8. One way to present the results is by using the horizontal axis to represent the different objective functions and the vertical axis for their numerical results. For each weight, the values of the objective functions are plotted and joined by line segments, creating a piecewise linear representation of the weights selected. To better present the data graphically, we have normalized the results of the nine cases considered. Since the Smart Growth problem is a multiobjective minimization, we have scaled the results of all objective functions to be in the range [0,1], where 0 is the most desirable value and 1 is the least desirable value. Using this scale, the maximum profit has a zero value and the minimum profit has a value of one. Values closer to zero are thus more desirable for all perspectives.

Table 7
Number of units or acres developed by each perspective in each zone, lower and upper bounds

Case	Description	Units RLD	Units RMD	Units RHD	Area commercial km^2 (mi^2)	Area industrial km^2 (mi^2)
1	Planner alone	1745	8190	4887	1.641 (0.634)	0.985 (0.380)
2	Planner Pareto	2331	12,285	6384	1.641 (0.634)	1.027 (0.397)
3	Environmentalist alone	1554	8190	4256	1.094 (0.423)	0.724 (0.279)
4	Environmentalist Pareto	1554	8190	4256	1.096 (0.423)	0.724 (0.279)
5	Conservationist alone	1554	8296	4687	1.481 (0.572)	1.027 (0.397)
6	Conservationist Pareto	2329	12,282	6372	1.638 (0.632)	1.027 (0.397)
7	Developer alone	2331	12,285	6384	1.641 (0.634)	1.027 (0.397)
8	Developer Pareto	2331	12,285	6381	1.641 (0.634)	1.027 (0.397)
9	All perspectives	2331	12,285	6384	1.641 (0.634)	0.740 (0.286)
	Lower bound	1554	8190	4256	1.094 (0.423)	0.724 (0.279)
	Upper bound	2331	12,285	6384	1.641 (0.634)	1.085 (0.419)
	Available	971	3359	1926	0.690 (0.266)	1.027 (0.397)
	Available from 99	7572	30,998	62,242	31.612 (12.205)	31.612 (12.205)
	Total available[a]	8543	34,357	64,168	32.301 (12.472)	32.639 (12.602)

[a]Assuming that all the available parcels from 99 go to each category indicated.

We have used Eqs. (21) and (22) to normalize the data

$$X_{\text{nor}} = \frac{X_i - \text{Min}_i\{X_i\}}{\text{Max}_i\{X_i\} - \text{Min}_i\{X_i\}}, \tag{21}$$

for those variables whose lower values are preferred to higher values (Compactness, Imperviousness Change, Environmentally Sensitive Area), and

$$Y_{\text{nor}} = \frac{\text{Max}_i\{Y_i\} - Y_i}{\text{Max}_i\{Y_i\} - \text{Min}_i\{Y_i\}} \tag{22}$$

for those variables whose higher values are preferred than lower values (Profit).

The different solutions are labeled 1–9 in order to better track the relative values for each objective. Where two or more solutions converge on one point, we have used different line-types

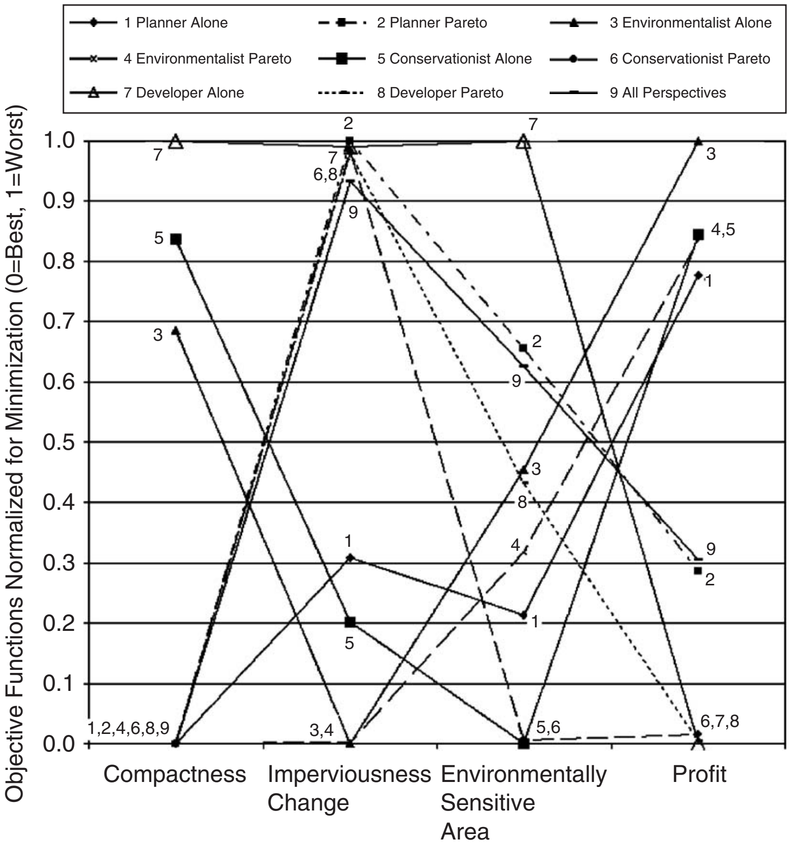

Fig. 9. Value path representation for all cases.

Table 8
Value path information

Case description		Normalized maximum distance squared compactness	Normalized imperviousness change	Normalized environmentally sensitive area	Normalized profit
1.	Planner alone	0.00	0.31	0.21	0.78
2.	Planner Pareto	0.00	1.00	0.66	0.29
3.	Environmentalist alone	0.69	0.00	0.45	1.00
4.	Environmentalist Pareto	0.00	0.00	0.32	0.84
5.	Conservationist alone	0.84	0.20	0.00	0.84
6.	Conservationist Pareto	0.00	0.98	0.00	0.01
7.	Developer alone	1.00	0.99	1.00	0.00
8.	Developer Pareto	0.00	0.98	0.43	0.00
9.	All perspectives	0.00	0.93	0.63	0.31

for clarity. One remarkable, although expected conclusion is that those solutions associated with the maximum profit (6—Conservationist Pareto, 7—Developer Alone, and 8—Developer Pareto) are associated with the highest level of imperviousness change. Also, those solutions with low imperviousness change have a low profit value.

The compactness and the imperviousness change and profit can be divided into two groups. In the case of compactness, the two groups are either zero (best possible case) or one of three values (0.7, 0.8 and 1). In the case of the imperviousness change, there are three low values (0, 0.2 and 0.3) and a group of high values (0.9, 1). The profit values are also separated, but to a lesser extent as compared to the other objectives.

4.4. Analysis of the compactness objective

Since the model in (20) considers compactness of each quadrant separately, it is convenient to analyze each individually. Fig. 10 presents details of each quadrant with two key rectangles drawn one inside the other. The inner rectangle surrounds all previously developed parcels and the outer rectangle encloses all the parcels in that quadrant, or the quadrant rectangle. Parcels that do not belong to the quadrant in question have been removed for clarity of presentation. Each quadrant has a different potential for compactness. For example, the inner rectangles for quadrants 2 and 4, enclosing all previously developed parcels, are almost the same as the outer rectangles, which include all the parcels. There is not much choice relative to compactness in terms of which new parcels to select for development. Conversely, quadrants 1 and 3 have more potential for compactness given their configuration of parcels that are already developed or available for development. Thus, a key ratio related to the efficiency of the compactness can be defined for each quadrant, i.e., the ratio of the squared diagonal of the inner rectangle to the squared diagonal of the quadrant rectangle. A lower value means a greater potential for more compactness.[9] Table 9 provides these ratios for each quadrant based on dividing key measures for the inner rectangle by the corresponding measures for the outer rectangle. These key measures are: the diagonal squared, the unsquared diagonal, and the area. Table 10 shows the ratio of the new inner rectangle (including parcels chosen to be developed) vs. the former inner rectangle (just previously developed parcels) for each case. A value of more than one means that the developed area has been expanded.

The results of these cases can also be presented using value paths, as shown in Fig. 11.

4.5. Policy implications

Currently, decision-making about land development is largely decentralized and primarily dependent on policies of local municipalities and counties. Meanwhile, the consequences can be more far-reaching. For instance, precious natural resources often span across political boundaries and land developers rarely consider any balance between the geographic demand for future growth and government policies that provide the best business climate for development. Given

[9]As far as we know, these compactness measures are unique to our paper. They were introduced merely to convey that certain quadrants might have a "better chance" of keeping development in a compact region. Planners could use these compactness measures as one of a set of criteria to guide them in planning decisions about land development.

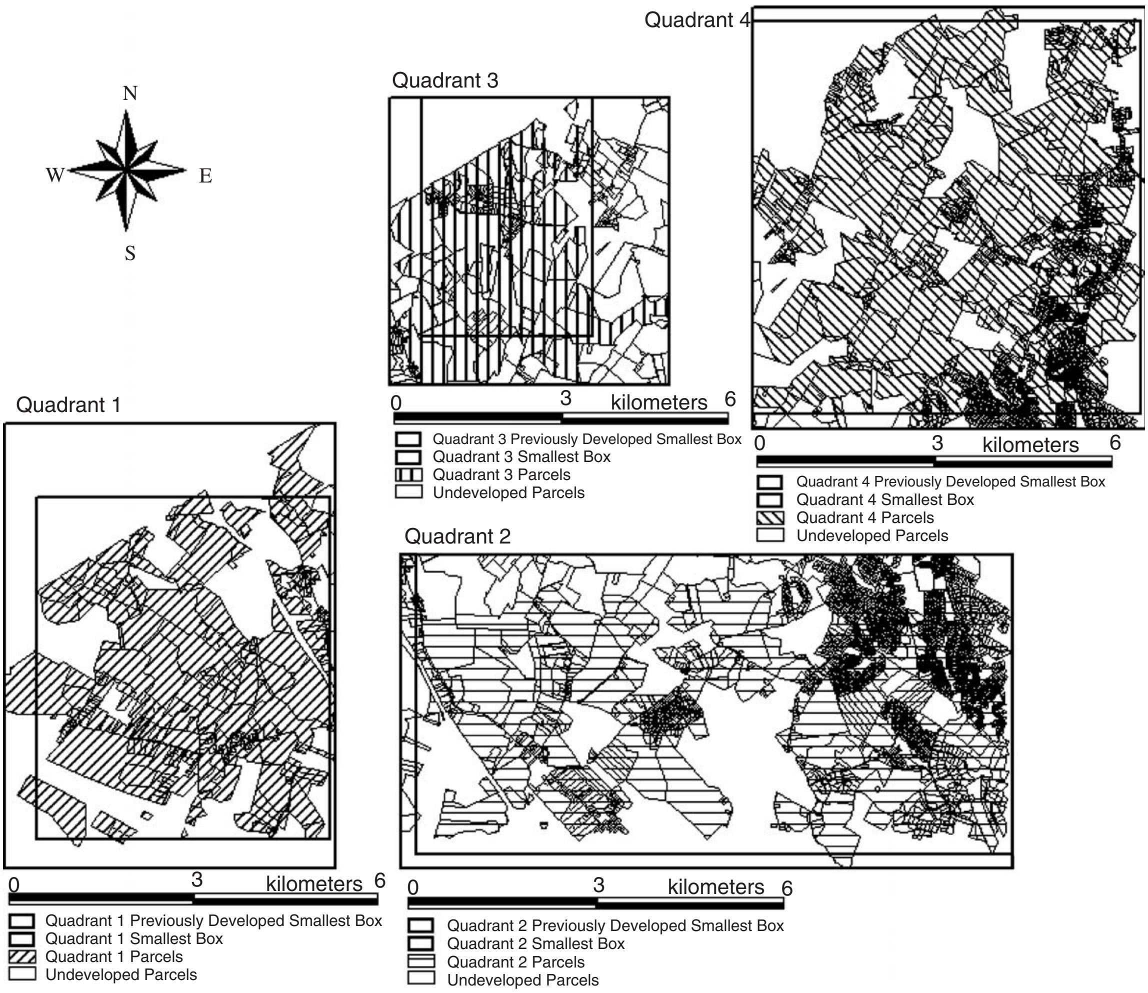

Fig. 10. Inner and quadrant rectangles for Quadrant 1 (left), Quadrant 2 (bottom right), Quadrant 3 (top center) and Quadrant 4 (top right).

Table 9
Compactness potential ratio calculation for each quadrant

Quadrant	Diagonal squared ratio	Diagonal ratio	Area ratio
1	0.70	0.84	0.72
2	0.98	0.99	0.96
3	0.54	0.73	0.51
4	0.91	0.96	0.92

these real dimensions to the land development problem, the ideas and results set forth in this paper suggest that Smart Growth might be viewed more broadly as a planning framework that can accept input from a wide spectrum of stakeholders.

Table 10
Compactness ratios for each quadrant based on the square of the diagonals

Case	Description	Q1 ratio	Q2 ratio	Q3 ratio	Q4 ratio
1	Planner alone	1.00	1.00	1.00	1.05
2	Planner Pareto	1.00	1.00	1.00	1.05
3	Environmentalist alone	1.39	1.00	1.43	1.05
4	Environmentalist Pareto	1.00	1.00	1.00	1.05
5	Conservationist alone	1.42	1.02	1.43	1.09
6	Conservationist Pareto	1.00	1.00	1.00	1.05
7	Developer alone	1.38	1.00	1.86	1.09
8	Developer Pareto	1.00	1.00	1.00	1.05
9	All perspectives	1.00	1.00	1.00	1.05

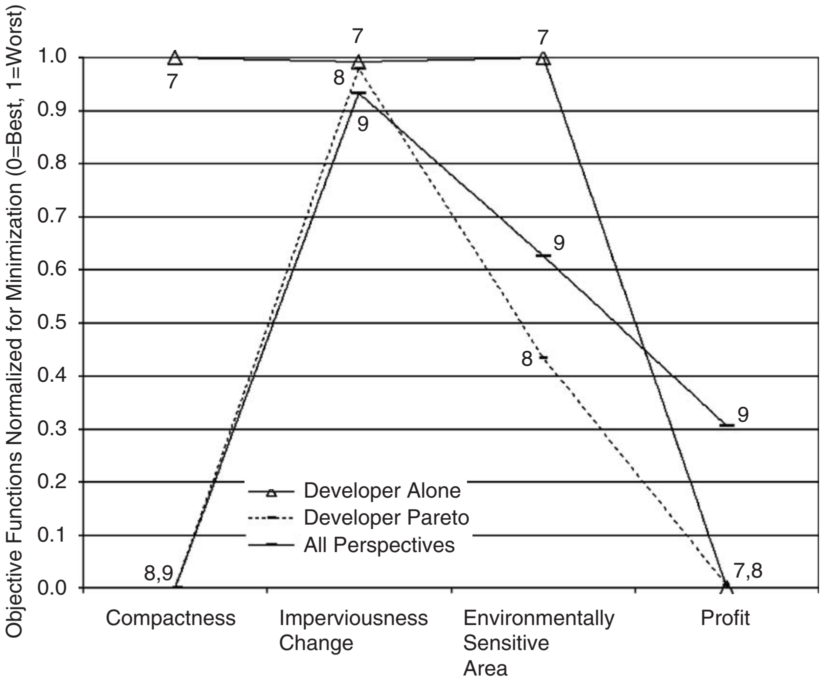

Fig. 11. Value path representation for the developer cases.

The tools, rather than the specific results of this paper, suggest a new approach available for policy-makers to address Smart Growth. There are two ways that these tools might play an important role in future land development decisions. First, the planner or policy maker could use this approach to predict future development outcomes from hypothetical new policies or land development related programs. The planner would need to quantitatively express the implications of a possible new policy in terms of its impacts on existing objective functions or constraints. Pareto optimal land development decisions would allow the policy maker to quantitatively and explicitly examine and critique estimated future growth. With this tool, the planner could refine potential new policies or programs in an iterative way until the predicted outcome matches the planner's intent. Second, planners can use these tools for arbitration. In this case, the tool could be used in real time at a meeting or hearing that brings together the range of interested stakeholders; however, prepared scenarios might need to be run ahead of time, given the computational complexity of solving problems in real time. Stakeholders could be surveyed on their values and desires and have these views translated into this multiobjective framework at that meeting. An iterative approach would be in order as the different parties refine and negotiate their views. The tool would aid in the simulation and visualization of these negotiated outcomes, providing a common language for different parties to exchange ideas and ensuring that decisions are being made objectively and optimally among all parties.

5. Conclusions

In this paper we have presented a multiobjective optimization formulation for Smart Growth in land development based on recognizing the objectives of four different types of stakeholders: the government planner, the environmentalist, the conservationist, and the land developer. This paper presented potential objective functions that might be posed by these various stakeholders. The resulting model was applied in the context of an illustrative example for a GIS-based data set for Montgomery County, Maryland.

This model had both linear and quadratic objective functions subject to linear and binary constraints. Using the weighting method [24] for determining Pareto optimal points resulted in QMIPs to be solved for each choice of positive weights applied to the stakeholder objective functions. The quadratic objective resulted from considering compactness of the developed area and represented the government planner's perspective. While other researchers have considered alternative formulations for compactness, our choice is advantageous since it represents a computationally attractive approach to model efficient infrastructure development. The weighted problems are convex QMIPs so that their relaxed versions, solved as part of the integer programming solution methodology, ensure that local solutions are global ones. Combined with a state-of-the-art solver for QMIPs, we have been able to solve rather large instances of these problems with some 3500 variables (mostly binary) and over 23,000 constraints. To illustrate the tradeoffs between stakeholders' individual objectives, we have considered nine different sets of weights and provided an analysis of the results.

This paper demonstrates the value of applying concepts of multiobjective optimization to the complex problem of Smart Growth and land use planning. The specific stakeholders identified and their proposed objective functions, while reasonable, are intended merely to illustrate how these concepts can be applied to this problem. The framework shown here can easily be modified to include other stakeholders' views or different objective functions. This process necessitates all those involved in the decision-making process to formulate explicit and quantifiable descriptions of their goals and constraints. Such formulations could serve to streamline discussions between different parties with a stake in the future development of a county, state, or region.

This paper also demonstrated the value of GIS technology that involves a geographic component in addressing decision making. The GIS was used at the front-end of this analysis to derive and store the quantities that were the focus of each of the stakeholders' objectives, as well as many of the constraints. Further, after optimizations were completed, the GIS provided a visual presentation of the alternative outcomes associated with the nine illustrative scenarios that were considered.

Also presented were several mathematical results concerning both the existence of a solution to this multiobjective optimization problem as well as the convexity of the QMIP weighting problems solved.

Acknowledgments

The authors gratefully acknowledge support received to perform this study from the National Center for Smart Growth Research and Education at the University of Maryland, College Park, Maryland. The authors also thank the Maryland Department of Planning for the use of their MdProperty View and Land Use Land Cover data and Professors Ben Hobbs and Charles ReVelle of Johns Hopkins University for their useful comments. Finally, we thank the editor and the anonymous reviewers for their insightful comments.

Appendix

The following assumption is needed for later analysis.

Assumption 1. For each parcel i,

(a) $\text{row}_\text{N}(i) > \text{rows}_\text{S}(i)$, i.e., the parcel has a positive height,
(b) $\text{col}_\text{E}(i) > \text{col}_\text{W}(i)$, i.e., the parcel has a positive width.

A.1. Feasibility

The feasible region to the multiobjective model (20) may be empty. There are many reasons why this problem may be infeasible. We illustrate with two cases. For example, when the constants $\min_\text{RLD}$ and $\max_\text{RLD}$ are chosen so that $\sum_{i\in S_{11}} \text{units}_i + \sum_{i\in S_{99}} \text{units}_i < \min_\text{RLD} < \max_\text{RLD}$.

(17a) can never be satisfied since

$$\sum_{i\in S_{11}} \text{units}_i\, d_i + \sum_{i\in S_{99}} \text{units}_i\, \text{RLD}_i \leqslant \sum_{i\in S_{11}} \text{units}_i + \sum_{i\in S_{99}} \text{units}_i < \min_\text{RLD} < \max_\text{RLD}.$$

Another case is when $\min_\text{RLD} < \max_\text{RLD} < \sum_{i\in S_{11}} \text{units}_i$ there are more units in S_{11} than are actually needed, i.e., $\max_\text{RLD} < \sum_{i\in S_{11}} \text{units}_i$. Thus, some parcels should not be developed. Moreover, none of the unassigned parcels need to be developed.

The main question is how to identify which parcels will remain undeveloped. Due to the binary nature of the development variables d_i, the function $\sum_{i\in S_{11}} \text{units}_i\, d_i$ is "lumpy". This feature makes the following infeasibility possible: Suppose that the lower and upper bounds on the number of units are $\min_\text{RLD} = 380$, $\max_\text{RLD} = 400$ but that $\sum_{i\in S_{11}} \text{units}_i = 415$. To satisfy (17a), we would want to find a set of parcels $D \subseteq S_{11}$ such that $\sum_{j\in D} \text{units}_j \in [15, 35]$ and then set $d_j = 0$, $j \in D$, $d_i = 1$, $\forall i \in S_{11} - D$. This result would not be possible if the smallest number of units for any parcel in

S_{11} were larger than 35. If we designate the parcel in S_{11} with the smallest number of units as j, this result is clear since for any D such that $\{j\} \subseteq D \subseteq S_{11}$, $\sum_{i \in S_{11}-D} \text{units}_i \leqslant \sum_{i \in S_{11}-\{j\}} \text{units}_i < \min_{\text{RLD}} < \max_{\text{RLD}}$. A natural question is whether (17a) can be satisfied with some units from the S_{99} pool of unassigned parcels in this case. The answer is no since by (18a), $\sum_{i \in S_{11}} \text{units}_i > \max_{\text{RLD}} > \min_{\text{RLD}}$, which forces $y_{\text{RLD}} = 0$ (otherwise a contradiction), which in turn forces $\text{RLD}_i = 0$, $\forall i \in S_{99}$ via the other part of (18a), namely, $\sum_{i \in S_{99}} \text{RLD}_i \, \text{units}_i \leqslant M y_{\text{RLD}}$. Thus, in this case there is also no feasible solution to (20) because of the "lumpiness" of the data.

Theorem 1. *The objective to the weighted problem for* (20) *is convex in its variables as long as the weights* $w_1, w_2, w_3, w_4 \geqslant 0$.

Proof. Consider the function $f(r_{\text{N},q}, r_{\text{S},q}, c_{\text{E},q}, c_{\text{W},q}) = (r_{\text{N},q} - r_{\text{S},q})^2 + (c_{\text{E},q} - c_{\text{W},q})^2$. This function is convex in the variables $r_{\text{N},q}, r_{\text{S},q}, c_{\text{E},q}, c_{\text{W},q}$ since its Hessian matrix

$$H_q = \begin{pmatrix} 2 & -2 & 0 & 0 \\ -2 & 2 & 0 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & -2 & 2 \end{pmatrix}$$

has eigenvalues $\{0, 0, 4, 4\}$ so that it is (symmetric) positive semi-definite, or equivalently that f is convex [33]. The weighted objective function is the positive sum of convex functions. □

Theorem 2. *Assuming the weighted problem to* (20) *has an optimal solution and Assumption* 1 *holds, then the constraints* (9a)–(9d) *ensure that* $r_{\text{N},q}, r_{\text{S},q}, c_{\text{E},q}, c_{\text{W},q}$ *correspond, respectively, to the northernmost, southernmost, easternmost, and westernmost borders of all the developed parcels in subdivision* q *for* $q = 1, \ldots, Q$.

Proof. Let $x^* = \{d_i^* \forall i \in S, \text{RLD}_i^*, \text{RMD}_i^*, \text{RHD}_i^*, \text{COM}_i^*, \text{IND}_i^*, \forall i \in S_{99}, y^*_{\text{RLD}}, y^*_{\text{RMD}}, y^*_{\text{RHD}}, y^*_{\text{COM}}, y^*_{\text{IND}}, w^*_{\text{RLD}}, w^*_{\text{RMD}}, w^*_{\text{RHD}}, w^*_{\text{COM}}, w^*_{\text{IND}}, (r_{\text{N},q})^*, (r_{\text{S},q})^*, (c_{\text{E},q})^*, (c_{\text{W},q})^*, q = 1, \ldots, Q\}$ be an optimal solution to the weighted problem to (20). There are two cases to consider.

For case 1, assume that for subdivision q there is at least one developed parcel in a solution. By (9a), (9b), and Assumption 1, we see that for a developed parcel j $(r_{\text{S},q})^* \leqslant \text{rows}_{\text{S}}(j) < \text{row}_{\text{N}}(j) \leqslant (r_{\text{N},q})^*$ so that $(r_{\text{N},q})^* - (r_{\text{S},q})^* > 0$. Suppose for sake of contradiction that for all indices i, (9b) holds as a strict inequality. Consider the feasible value for $r_{\text{N},q}$ of $\hat{r}_{\text{N},q} = (r_{\text{N},q})^* - \gamma$, where γ is sufficiently small and satisfies $0 < \gamma < 2(r_{\text{N},q})^* - (r_{\text{S},q})^*$ and all other values are the same as in x^*. Then we have

$$\begin{aligned} &[(\hat{r}_{\text{N},q}) - (r_{\text{S},q})^*]^2 + [(c_{\text{E},q})^* - (c_{\text{W},q})^*]^2 \\ &\quad = [((r_{\text{N},q})^* - \gamma) - (r_{\text{S},q})^*]^2 + [(c_{\text{E},q})^* - (c_{\text{W},q})^*]^2 \\ &\quad = [(r_{\text{N},q})^* - (r_{\text{S},q})^*]^2 + [(c_{\text{E},q})^* - (c_{\text{W},q})^*]^2 - 2[(r_{\text{N},q})^* - (r_{\text{S},q})^*]\gamma + \gamma^2 \\ &\quad < [(r_{\text{N},q})^* - (r_{\text{S},q})^*]^2 + [(c_{\text{E},q})^* - (c_{\text{W},q})^*]^2 \end{aligned}$$

as long as the function $\theta(\gamma) = -2[(r_{N,q})^* - (r_{S,q})^*]\gamma + \gamma^2 < 0$. This result is guaranteed since $\theta(\gamma)$ has roots at $\{0, 2((r_{N,q})^* - (r_{S,q})^*)\}$ and is negative in between these roots. Thus, there is a contradiction to the optimality of x^* showing that there must be an index i for this subdivision since (9b) holds as an equality. Similar reasoning applies to (9a), (9c), and (9d) ensuring the desired result.

For case 2, assume that for the subdivision q, no parcels are developed in an optimal solution x^*. In this case, the northernmost, southernmost, easternmost, and westernmost borders are arbitrary since the set of developed parcels is vacuous. However, to make sense, we must have $(r_{N,q})^* \geqslant (r_{S,q})^*$, $(c_{E,q})^* \geqslant (c_{W,q})$. But (9a)–(9d) show that

$$r_{S,q} \leqslant \mathrm{rows}_S(i) + M \quad \text{and} \quad \mathrm{row}_N(i) - M \leqslant r_{N,q},$$
$$c_{W,q} \leqslant \mathrm{col}_W(i) + M \quad \text{and} \quad \mathrm{col}_E(i) - M \leqslant c_{E,q}$$

which, in conjunction with the other constraints, allows for any ordering between the pairs of variables $[(r_{N,q})^*, (r_{S,q})^*]$, and $[(c_{E,q})^*, (c_{W,q})]$, given that M is a sufficiently large positive value. Hence, by an optimality argument, it must be the case that $(r_{N,q})^* = (r_{S,q})^*$, $(c_{E,q})^* = (c_{W,q})^*$ to minimize the objective function term for this subdivision q. Such values make sense in light of the vacuous set of developed parcels for the subdivision. $\square$

References

[1] Gillham O. The limitless city: a primer on the urban sprawl debate. Washington, DC: Island Press; 2002.
[2] O'Neill D. Smart growth, myth and fact. Washington, DC: Urban Land Institute; 1999.
[3] Downs A. What does smart growth really mean? Planning 2001;67(4).
[4] Bengston DN, Fletcher JO, Nelson KC. Public policies for managing urban growth and protecting open space: policy instruments and lessons learned in the United States. Landscape and Urban Planning 2004;69:271–86.
[5] Miller JS, Hoel LA. The "smart growth" debate: best practices for urban transportation planning. Socio-Economic Planning Sciences 2002;36:1–24.
[6] Arendt R. Linked landscapes creating greenway corridors through conservation subdivision design strategies in the northeastern and central United States. Landscape and Urban Planning 2004;68:241–69.
[7] Conine A, Xiang W-N, Young J, Whitley D. Planning for multi-purpose greenways in Concord, North Carolina. Landscape and Urban Planning 2004;68:271–87.
[8] Frenkel A. The potential effect of national growth-management policy on urban sprawl and the depletion of open spaces and farmland. Land Use Policy 2003, in press.
[9] Moglen G, Gabriel SA, Faria JA. A framework for quantitative smart growth in land development. Journal of the American Water Resources 2003;39(4):947–59.
[10] Batty M. Urban evolution on the desktop: simulation with the use of extended cellular automata. Environment Planning A 1997;30(11):1943–68.
[11] Miettinen KM. Nonlinear multiobjective optimization. Boston: Kluwer Academic Publishers; 1999.
[12] Steuer RE. Multiple criteria optimization: theory, computation, and application. New York: Wiley; 1986.
[13] Alig RJ, Kine JD, Lichtenstein M. Urbanization on the US landscape: looking ahead in the 21st century. Landscape and Urban Planning 2004;69(2–3):219–34.
[14] Reilly J, Maggio P, Karp S. A model to predict imperviousness surface for regional and municipal land use planning purposes. Environmental Impact Assessment Review 2004;24:363–82.
[15] Preuss I, Vemuri AW. "Smart growth" and dynamic modeling: implications for quality of life in Montgomery County, Maryland. Ecological Modelling 2004;171:415–32.
[16] Prenzel B. Remote sensing-based quantification of land-cover and land-use change for planning. Progress in Planning 2004;61(4):281–99.

[17] Bammi D, Bammi D. Land use planning: an optimizing model. OMEGA—The International Journal of Management Science 1975;3(5):583–93.
[18] Bammi D, Bammi D. Development of comprehensive land use plan by means of a multiple objective mathematical programming model. Interfaces 1979;9(2):50–63.
[19] Wright J, ReVelle C, Cohon J. A multiobjective integer programming model for the land acquisition problem. Regional Science and Urban Economics 1983;13:31–53.
[20] ReVelle C, McGarity AE, editors. Design and operation of civil and environmental engineering systems. New York: Wiley; 1997.
[21] Benabdallah S, Wright JR. Multiple subregion allocation models. Journal of Urban Planning and Development 1992;118(1):24–40.
[22] Gilbert KC, Holmes DD, Rosenthal RE. A multiobjective discrete optimization model for land allocation. Management Science 1985;31(12):1509–22.
[23] Beinat E, Nijkamp P. Multicriteria analysis for land-use management. Dordrecht: Kluwer Academic Publishers; 1998.
[24] Cohon JL. Multiobjective programming and planning. New York: Academic Press; 1978.
[25] Mansini R, Speranza MG. Heuristic algorithms for the portfolio selection problem with minimum transaction lots. European Journal of Operational Research 1999;114:219–33.
[26] Schueler TR, Holland TR, editors. The practice of watershed protection. Ellicott City, MD: The Center for Watershed Protection; 2000.
[27] Deb K. Multi-objective optimization using evolutionary algorithms. Sussex, England: Wiley, West; 2001.
[28] Arnold CL, Gibbons CJ. Impervious surface coverage: the emergence of a key environmental indicator. American Planning Association Journal 1996;62(2):243–58.
[29] Schueler TR. The importance of imperviousness. Watershed Protection Techniques 1994;1(3):100–11.
[30] Leopold LB. Hydrology for urban planning: a guidebook on the hydrologic effects of land use. US Geological Surve Circular 554, 1968; 18pp.
[31] Paul MJ, Meyer JL. Streams in the urban landscape. Annual Review of Ecology and Systematics 2001;32:333–65.
[32] US Geological Survey. National Land Cover Characterization 2001, available at: http://landcover.usgs.gov/natlandcover_2000.asp, Sioux Falls, SD, 2004.
[33] Bazaraa MS, Sherali HD, Shetty CM. Nonlinear programming theory and algorithms. New York: Wiley; 1979.
[34] Maryland Department of Planning. MDProperty view, 2000 edition. Baltimore, MD: Planning Data Services; 2000.
[35] Soil Conservation Service. National engineering handbook, Supplement A, Section 4, Hydrology. Washington, DC: US Department of Agriculture; 1985 [Chapter 10].
[36] Daskin MS. Network and discrete location; models, algorithms and applications. New York: Wiley; 1995.
[37] Saaty TL. The analytic hierarchy process. New York: McGraw Hill; 1980.

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